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Recursive filtering for complex networks with time-correlated fading channels: An outlier-resistant approach

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ABSTRACT

In this paper, the outlier-resistant recursive filtering problem is fully discussed for complex networks with time-correlated fading channels. Each sensor is able to communicate with its corresponding filter within a set of time-correlated fading channels, and the channel coefficient is assumed to be governed by certain dynamical process. In order to alleviate undesired effects (e.g. performance degradation or even divergence of the filtering error) from possible measurement outliers, a certain saturation structure is introduced in our constructed filter. The purpose is at estimating network states with satisfactory error dynamics with not only time-correlated fading channels but also measurement outliers. First, an augmented model is constructed in order to combine network dynamic evolutions along with channel coefficients. Subsequently, by means of the inductive method, upper covariance bounds are first given and later minimized by properly parameterizing filter gains. Finally, two example are given for effectiveness validation.

1. Introduction

Complex networks (CNs) have been extensively investigated due to their distinctive capacities of describing a variety of real-world coupled systems with examples including worldwide networks, nervous system, social relationship networks, power grids and transportation networks [2,8,32]. Simply put, a classical CN is a kind of large-scale networks comprising a family of nodes which are interconnected with each other through a known topology structure. More specifically, each node in the CN represents a particular subsystem/unit having its own physical meaning, whose dynamics can be depicted by a certain dynamical equation that mainly contains two parts: the isolated node dynamics and the coupling dynamics. Up to now, the analysis of the dynamics of CNs has attracted much research enthusiasm from many research communities and, accordingly, excellent works have been extensively reported with focus on synchronization, stability, consensus, pinning control and estimation for CNs, see e.g. [12,21,39,31,11,38,24,29].

Among the dynamics analysis issues, the estimation problem for CNs has long been playing a crucial role in understanding the network structure and achieving certain tasks. Filtering is able to generate estimates for underlying systems by means of using available measurements. In recent years, several filter design algorithms have been put forward to cater

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for various performance requirements, see [40,7] for recursive filters, [17,15,36] for H_∞ filters, [9,26] for dissipative filters, [33,5,41] for set-membership filters, and [35,42] for finite-time filters. Most of above results have been obtained via an augmentation approach by formulating all the states of the network into a single augmented vector. However, due to the large size of CNs, this augmentation technique would inevitably lead to a significant increase of the computation complexity. In this case, many researchers have recently attempted to avoid the computation burden arising from augmentations [19].

When realizing the filtering algorithm of CNs, the sensors always transmit the measurement outputs to remote filters through wireless network. As is well known, the wireless network is often subject to the influence of channel fading resulting from the path loss, shadowing and multipath propagation. By definition, fading can be understood as a phenomenon that the amplitude/phase of the signal undergo random variations during the transmission. Such variations, if not dealt with properly, would seriously degrade the system performance and dedicated attention has recently been directed to certain analysis/design problem subject to fading channels, see e.g. [28,27,37]. Noticing that the existing studies have based themselves on an implicit assumption that the fading coefficient is mutually uncorrelated at different time instants. Unfortunately, this assumption goes against the reality since the fading coefficient may exhibit time-correlated property due to the memorability of channels. In this regard, the time-correlated fading channels (TcFCs) have become a focus of research with a few initial results available [20,4,16,14,34]. For example, in [34], the TcFCs have been introduced into the design of recursive filters for an uncertain system.

In practice, measurements are delivered from the sensor to the filter through bandwidth-limited channels and, owing to unavoidable environment changes and malicious cyber-attacks, such measurements might suffer from abnormal yet large-amplitude disturbances, i.e. measurement outliers. Obviously, the outliers deviate significantly from the true signals and any estimation based such outlier-contaminated measurements would be unreliable. Therefore, there has been certain growing interest in mitigating undesired effects from the measurement outliers. For example, a stubborn observer has been given in [1] to complete estimation task subject to measurement outliers, where a saturated output injection has been introduced to restrain the impacts from possible outliers. Following this idea, the results developed in [1] have been extended to the cases of multirate systems[30], neural networks [18] and sensor networks[23]. Unfortunately, filtering results on CNs with TcFCs have been very few.

In this paper, we intend to investigate outlier-resistant recursive filtering for CNs with TcFCs. This is non-trivial as we are facing three inevitable difficulties in 1) constructing a valid recursive filter capable of tolerating the impacts from possible measurement outliers; 2) examining the effects from TcFCs and measurement outliers on desired performance; and 3) parameterizing desired gains such that filtering error covariance (FEC) bounds are minimized. The contributions are: *1) a new recursive filtering problem is studied for CNs subject to both measurement outliers and TcFCs; 2) a novel outlier-resistant filter is constructed so as to make it robust to the measurement outliers; and 3) an effective filtering algorithm is developed to compute the gains of the preferred recursive filter.*

Notation. The operators $(\cdot)^{-1}, (\cdot)^T$ and $\mathrm{tr}\{\cdot\}$ represent, respectively, the inverse, the transpose and the trace. For a matrix $\Omega, \Omega > 0$ $(\Omega \geqslant 0)$ indicates that Ω is positive (semi-positive) and $\lambda_{\max}(\Omega)$ means the largest eigenvalue of Ω.

2. Problem formulation

Consider a complex network with Q coupled nodes, where the dynamics of node p is

$$x_{p,t+1} = A_{p,t}x_{p,t} + \sum_{q=1}^{Q} w_{pq}\Pi x_{q,t} + B_{p,t}\omega_t, \tag{1}$$

where $x_{p,t} \in \mathbb{R}^{n_x}$ $(p = 1, \ldots, Q)$ stands for the state of node p. $\mathscr{W} = (w_{pq})_{Q\times Q}$ is the topology configuration matrix and $w_{pq} > 0$ if node p could receive information from node q, otherwise $w_{pq} = 0$. Also, $\mathscr{W}$ is symmetric with $w_{pp} = -\sum_{q=1,q\neq p}^{Q} w_{pq}$. $\Pi = \mathrm{diag}\{\varsigma_1, \varsigma_2, \ldots, \varsigma_{n_x}\} \in \mathbb{R}^{n_x\times n_x}$ is an inner-coupling matrix. $A_{p,t}$ and $B_{p,t}$ are known real-valued matrices. $\omega_t \in \mathbb{R}^{n_w}$ is the Gaussian noise with zero-mean and covariance $\mathscr{R}_t > 0$. $x_{p,0}$ is random having mean $\bar{x}_p$ and covariance $\tilde{x}_p > 0$.

For node p, the measured output is expressed by

$$y_{p,t} = C_{p,t}x_{p,t} + E_{p,t}\upsilon_t, \tag{2}$$

where $y_{p,t} \in \mathbb{R}^{n_y}$ is the measurement output of node $p, C_{p,t}$ and $E_{p,t}$ are known matrices, and $\upsilon_t \in \mathbb{R}^{n_\upsilon}$ is the Gaussian distributed measurement noise with zero-mean and covariance $\mathscr{Q}_t > 0$.

In this paper, we consider that the measured output $y_{p,t}$ is transmitted through a TcFC. For sensor p, denote $\bar{y}_{p,t}$ as the actually received measurement after transmitting to its corresponding filter under TcFCs, which can be described as

$$\bar{y}_{p,t} \triangleq \tau_{p,t}y_{p,t} \tag{3}$$

where $\tau_{p,t} \in \mathbb{R}$ is the coefficient of fading channel satisfying

$$\tau_{p,t+1} = \sqrt{\lambda_{p,t}}\tau_{p,t} + \sqrt{1-\lambda_{p,t}}v_{p,t}. \tag{4}$$

Here, $\lambda_{p,t} \in (0,1)$ is the time-correlation factor, $v_{p,t}$ and $\tau_{p,0}$ are two Gaussian distributed stochastic variables with $\mathbb{E}\{v_{p,t}\} = 0, \mathbb{E}\{\tau_{p,0}\} = \bar{v}_p$ and $\mathbb{E}\left\{v_{p,t}^2\right\} = \mathbb{E}\left\{(\tau_{p,0} - \bar{v}_p)^2\right\} = \tilde{v}_p$.

Remark 1. In practice, when the measurement signal is propagated through a wireless channel, its amplitude/phase might experience fading caused by the interference from the real environment. In this paper, TcFC (3)–(4) is utilized to account for fading. It is easily seen that the fading intensity is depicted by coefficient $\tau_{p,t}$ that evolves according to dynamics (4). Note that such a model could reveal the time-correlated nature of the channel coefficient, which is fairly common in reality, see e.g. [34,14].

Assumption 1. All random variables $x_{p,0}, \omega_t, \upsilon_t, \tau_{p,0}$ and $v_{p,t}$ are mutually uncorrelated.

Remark 2. In Assumption 1, it is assumed that all the mentioned stochastic variables are mutually uncorrelated. Indeed, this assumption would limit the application scope of the proposed design method when these stochastic variables are correlated with each other. However, in many real-world applications, this assumption we have made is fairly standard since these stochastic variables have different random sources. For instance, the process noise ω_t accounts for a type of disturbances that can influence the network dynamics. The measurement noise υ_t describes the disturbance occurred when sensors collect the information from system. The channel noise $v_{p,t}$ arises from the data transmission between the sensor and the filter through fading channels. Note that such an assumption has been frequently used in both theoretical research and engineering practice.

Letting $\varphi_{p,t} \triangleq \tau_{p,t}x_{p,t}$, it follows from (1) and (4) that

$$\begin{aligned}\varphi_{p,t+1} = {} & \sqrt{\lambda_{p,t}}(A_{p,t} + w_{pp}\Pi)\varphi_{p,t} + \sqrt{\lambda_{p,t}}\tau_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq} \\ & \times\Pi x_{q,t} + \sqrt{\lambda_{p,t}}\tau_{p,t}B_{p,t}\omega_t + \sqrt{1-\lambda_{p,t}}v_{p,t} \\ & \times(A_{p,t} + w_{pp}\Pi)x_{p,t} + \sqrt{1-\lambda_{p,t}}v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq} \\ & \times\Pi x_{q,t} + \sqrt{1-\lambda_{p,t}}v_{p,t}B_{p,t}\omega_t.\end{aligned} \tag{5}$$

Setting $\mathscr{X}_{p,t} \triangleq \begin{bmatrix} x_{p,t}^T & \varphi_{p,t}^T \end{bmatrix}^T$, the following augmented system is derived:

$$\begin{aligned}\mathscr{X}_{p,t+1} = {} & \mathscr{A}_{1p,t}\mathscr{X}_{p,t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t \\ & + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} + v_{p,t}\mathscr{B}_{2p,t}\omega_t\end{aligned} \tag{6}$$

with

$$\bar{y}_{p,t} = \bar{C}_{p,t}\mathscr{X}_{p,t} + \tau_{p,t}E_{p,t}\upsilon_t \tag{7}$$

where

$$\begin{aligned}& \mathscr{A}_{1p,t} \triangleq diag\left\{A_{p,t} + w_{pp}\Pi, \sqrt{\lambda_{p,t}}(A_{p,t} + w_{pp}\Pi)\right\}, \mathscr{B}_{1p,t} \triangleq diag\left\{B_{p,t}, \sqrt{\lambda_{p,t}}B_{p,t}\right\}, \\ & \tilde{\tau}_{1p,t} \triangleq diag\{I, \tau_{p,t}I\}, \bar{\Pi}_{1p,t} \triangleq \begin{bmatrix} \Pi & 0 \\ \sqrt{\lambda_{p,t}}\Pi & 0 \end{bmatrix}, \bar{\Pi}_{2p,t} \triangleq \begin{bmatrix} 0 & 0 \\ \sqrt{1-\lambda_{p,t}}\Pi & 0 \end{bmatrix}, \\ & \tilde{\tau}_{2p,t} \triangleq \begin{bmatrix} I \\ \tau_{p,t}I \end{bmatrix}, \mathscr{A}_{2p,t} \triangleq \begin{bmatrix} 0 & 0 \\ \sqrt{1-\lambda_{p,t}}(A_{p,t} + w_{pp}\Pi) & 0 \end{bmatrix}, \\ & B_{2p,t} \triangleq \begin{bmatrix} 0 \\ \sqrt{1-\lambda_{p,t}}B_{p,t} \end{bmatrix}, \bar{C}_{p,t} \triangleq \begin{bmatrix} 0 & C_{p,t} \end{bmatrix}.\end{aligned}$$

For node p, based on the received fading measurement $\bar{y}_{p,t+1}$, an outlier-resistant filter is designed.

$$\begin{aligned} \widehat{\mathscr{X}}_{p,t+1|t} &= \mathscr{A}_{1p,t}\widehat{\mathscr{X}}_{p,t|t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widehat{\mathscr{X}}_{q,t|t} \\ \widehat{\mathscr{X}}_{p,t+1|t+1} &= \widehat{\mathscr{X}}_{p,t+1|t} + \mathscr{K}_{p,t+1}\sigma_p\left(\Theta_{p,t+1}\right) \end{aligned} \tag{8}$$

where $\widehat{\mathscr{X}}_{p,t+1|t}$ and $\widehat{\mathscr{X}}_{p,t|t}$ are predicted and estimated values of $\mathscr{X}_{p,t}$ with $\widehat{\mathscr{X}}_{p,0|0} = \left[\overline{\mathscr{X}}_p^T \quad \bar{v}_p\overline{\mathscr{X}}_p^T\right]^T$. $\mathscr{K}_{p,t+1}$ is the gain. $\Theta_{p,t+1} \triangleq \bar{y}_{p,t+1} - \bar{C}_{p,t+1}\widehat{\mathscr{X}}_{p,t+1|t}$ is the innovation sequence. $\bar{\tau}_{p,t} \triangleq \operatorname{diag}\{I, \check{\tau}_{p,t}I\}$ with $\check{\tau}_{p,t} \triangleq \mathbb{E}\{\tau_{p,t}\}$ given later.

The saturated function $\sigma_p(\cdot) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ is defined as follows:

$$\sigma_p(\mu_t) = \left[\sigma_p^1(\mu_t^1) \quad \cdots \quad \sigma_p^{n_y}(\mu_t^{n_y})\right]^T, \quad \forall \mu_t \in \mathbb{R}^{n_y} \tag{9}$$

with $\sigma_p^l(\mu_t^l) \triangleq \operatorname{sign}(\mu_t^l)\min\{\delta_{p,t}, |\mu_t^l|\}$, where 'sign' is the signum function, μ_t^l denotes the lth element of vector μ_t, and $\delta_{p,t}$ stands for the saturation level satisfying

$$\delta_{p,t+1} = \gamma_p\delta_{p,t} + \epsilon_p\|\bar{y}_{p,t} - \bar{C}_{p,t}\widehat{\mathscr{X}}_{p,t|t-1}\| \tag{10}$$

where $\gamma_p \in [0,1), \epsilon_p > 0$ is a given weighting parameter and $\delta_{p,0} \geqslant 0$ is the given initial value.

Remark 3. In the constructed filter (8), a saturated function $\sigma_p(\cdot)$ is put forward by specifying a certain bound on the innovation with hope to restrain the effects of possible measurement outliers. Compared to the fixed saturation level used in [18,30], the saturation level proposed in (10) is a dynamic variable that is adjusted in a dynamical way according to the innovation information. More precisely, if the norm of the innovation is increasing, then the saturation level would start increasing as well so as to relax the constraint the on innovation, and vise verse.

Defining $\widetilde{\mathscr{X}}_{p,t+1|t} \triangleq \mathscr{X}_{p,t+1} - \widehat{\mathscr{X}}_{p,t+1|t}$ as the prediction error and $\widetilde{\mathscr{X}}_{p,t+1|t+1} \triangleq \mathscr{X}_{p,t+1} - \widehat{\mathscr{X}}_{p,t+1|t+1}$ as the filtering error, we obtain from (6) and (8) that

$$\begin{aligned} \widetilde{\mathscr{X}}_{p,t+1|t} &= \mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t} \\ &\quad + \mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} + v_{p,t}\mathscr{B}_{2p,t}\omega_t \\ \widetilde{\mathscr{X}}_{p,t+1|t+1} &= \widetilde{\mathscr{X}}_{p,t+1|t} - \mathscr{K}_{p,t+1}\sigma_p\left(\Theta_{p,t+1}\right) \end{aligned} \tag{11}$$

where $\vec{\tau}_{p,t} \triangleq \operatorname{diag}\{0, (\tau_{p,t} - \check{\tau}_{p,t})I\}$.

Let $\mathscr{S}_{p,t|t} \triangleq \mathbb{E}\left\{\widetilde{\mathscr{X}}_{p,t|t}\widetilde{\mathscr{X}}_{p,t|t}^T\right\}$ be the FEC. Then, we aim to 1) seek an upper bound $\overline{\mathscr{S}}_{p,t|t}$ on the FEC $\mathscr{S}_{p,t|t}$ to ensure

$$\mathscr{S}_{p,t|t} \leqslant \overline{\mathscr{S}}_{p,t|t}$$

and 2) parameterize the gain matrix of the local filter (8) by minimizing such an upper bound.

Remark 4. In many recursive filtering problems, the FEC is usually seen as the most critical performance index and the main principle of traditional Kalman filtering is to minimize such a FEC. However, in this paper, we have taken into account the effects of measurement outliers and TcFCs in the filter design, which makes it almost impossible to calculate the exact FEC $\mathscr{S}_{p,t|t}$. Alternatively, we intend to find an upper bound $\overline{\mathscr{S}}_{p,t|t}$ for the FEC at each time-instant t such that $\mathscr{S}_{p,t|t} \leqslant \overline{\mathscr{S}}_{p,t|t}$.

3. Main results

To begin with, the saturation function $\sigma_p(\Theta_{p,t+1})$ in (11) is reformulated as follows in order to facilitate further analysis. From (9), it is easy to obtain

$$\sigma_p^l\left(\Theta_{p,t+1}^l\right) = \begin{cases} \Theta_{p,t+1}^l, & \text{if } |\Theta_{p,t+1}^l| \leqslant \delta_{p,t+1} \\ \operatorname{sign}\left(\Theta_{p,t+1}^l\right)\delta_{p,t+1}, & \text{otherwise} \end{cases} \tag{12}$$

where $\Theta_{p,t+1}^l$ is the lth element of $\Theta_{p,t+1}$.

Next, let's introduce a two-valued function $\rho(a,b)$ satisfying

$$\rho(a,b)=\begin{cases}0, & \text{if } a\leqslant b\\ 1, & \text{otherwise,}\end{cases} \tag{13}$$

which allows to rewrite (12) as

$$\begin{aligned}\sigma_p^l\left(\Theta_{p,t+1}^l\right)=&\left(1-\rho\left(|\Theta_{p,t+1}^l|,\delta_{p,t+1}\right)\right)\Theta_{p,t+1}^l\\&+\rho\left(|\Theta_{p,t+1}^l|,\delta_{p,t+1}\right)\mathrm{sign}\left(\Theta_{p,t+1}^l\right)\delta_{p,t+1}\end{aligned} \tag{14}$$

whose compact form can be expressed by

$$\sigma_p(\Theta_{p,t+1})=\quad(I-\Delta_{p,t+1})\Theta_{p,t+1}+\Delta_{p,t+1}\Lambda_{p,t+1} \tag{15}$$

with

$$\begin{aligned}\Delta_{p,t+1}&\triangleq\quad\mathrm{diag}\left\{\rho\left(|\Theta_{p,t+1}^1|,\delta_{p,t+1}\right),\ldots,\rho\left(|\Theta_{p,t+1}^{n_y}|,\delta_{p,t+1}\right)\right\},\\ \Lambda_{p,t+1}&\triangleq\quad\left[\mathrm{sign}\left(\Theta_{p,t+1}^1\right)\delta_{p,t+1}\quad\cdots\quad\mathrm{sign}\left(\Theta_{p,t+1}^{n_y}\right)\delta_{p,t+1}\right]^T.\end{aligned}$$

In terms of (15), the filtering error dynamics (11) can be rewritten as follows:

$$\begin{aligned}\widetilde{\mathscr{X}}_{p,t+1|t}=&\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t}+\sum_{q=1,q\neq p}^{Q}w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}+\sum_{q=1,q\neq p}^{Q}w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\\&+\mathscr{B}_{1p,t}\widetilde{\tau}_{2p,t}\omega_t+\nu_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t}+\nu_{p,t}\sum_{q=1,q\neq p}^{Q}w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}+\nu_{p,t}\mathscr{B}_{2p,t}\omega_t\\ \widetilde{\mathscr{X}}_{p,t+1|t+1}=&\left(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\right)\widetilde{\mathscr{X}}_{p,t+1|t}+\mathscr{K}_{p,t+1}\Delta_{p,t+1}\bar{C}_{p,t+1}\widetilde{\mathscr{X}}_{p,t+1|t}\\&-\mathscr{K}_{p,t+1}(I-\Delta_{p,t+1})\tau_{p,t+1}E_{p,t+1}\upsilon_{t+1}-\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}.\end{aligned} \tag{16}$$

In what follows, three lemmas are introduced that would be used in sequel.

Lemma 1. *Given real-valued matrices $\mathscr{U}$ and $\mathscr{V}$, the following inequality*

$$\mathscr{U}\mathscr{V}^T+\mathscr{V}\mathscr{U}^T\leqslant\alpha\mathscr{U}\mathscr{U}^T+\alpha^{-1}\mathscr{V}\mathscr{V}^T$$

holds for any positive scalar α.

Proof. Note that, for any positive scalar α, one has

$$\left(\alpha^{\frac{1}{2}}\mathscr{U}-\alpha^{-\frac{1}{2}}\mathscr{V}\right)\left(\alpha^{\frac{1}{2}}\mathscr{U}-\alpha^{-\frac{1}{2}}\mathscr{V}\right)^T\geqslant 0,$$

which implies

$$\mathscr{U}\mathscr{V}^T+\mathscr{V}\mathscr{U}^T\leqslant\alpha\mathscr{U}\mathscr{U}^T+\alpha^{-1}\mathscr{V}\mathscr{V}^T.$$

Then the proof of this lemma is complete. □

Lemma 2. *Denoting $\mathscr{T}_{p,t}\triangleq\mathbb{E}\left\{\tau_{p,t}^2\right\}$ and recalling the definition of $\check{\tau}_{p,t}$, the followings are obtained:*

$$\begin{aligned}\mathscr{T}_{p,t+1}&=\lambda_{p,t}\mathscr{T}_{p,t}+\left(1-\lambda_{p,t}\right)\widetilde{v}_p\\ \check{\tau}_{p,t+1}&=\sqrt{\lambda_{p,t}}\check{\tau}_{p,t}\end{aligned} \tag{17}$$

with $\mathscr{T}_{p,0}=\tilde{v}_p+\bar{v}_p^2$.

Proof. This proof follows readily from (4). □

Lemma 3. *Denote* $\mathfrak{X}_{p,t} \triangleq \mathbb{E}\left\{\mathscr{X}_{p,t}\mathscr{X}_{p,t}^T\right\}$. *Let scalars* $a_{p,t}, b_{p,t}$ *be given. Suppose*

$$\begin{aligned}\overline{\mathfrak{X}}_{p,t+1} = {} & (1+a_{p,t})\mathscr{A}_{1p,t}\overline{\mathfrak{X}}_{p,t}\mathscr{A}_{1p,t}^T + \left(1+a_{p,t}^{-1}\right)\bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{1p,t}^T\right)\Gamma_{1p,t} \\ & + \mathscr{B}_{1p,t}\Gamma_{2p,t}\mathscr{B}_{1p,t}^T + (1+b_{p,t})\tilde{v}_p\mathscr{A}_{2p,t}\overline{\mathfrak{X}}_{p,t}\mathscr{A}_{2p,t}^T + \tilde{v}_p\mathscr{B}_{2p,t}\mathscr{R}_t\mathscr{B}_{2p,t}^T \\ & + \left(1+b_{p,t}^{-1}\right)\tilde{v}_p\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{2p,t}^T \end{aligned} \tag{18}$$

holds with the initial condition $\overline{\mathfrak{X}}_{p,0} = \left[\bar{x}_p^2 + \tilde{x}_p \quad \bar{v}_p\left(\bar{x}_p^2 + \tilde{x}_p\right)\bar{v}_p\left(\bar{x}_p^2 + \tilde{x}_p\right) \quad \mathscr{T}_{p,0}\left(\bar{x}_p^2 + \tilde{x}_p\right)\right]$, *where*

$$\Gamma_{1p,t} = \mathrm{diag}\{I, \mathscr{T}_{p,t}I\},\ \bar{w}_p = \sum_{q=1,q\neq p}^{Q} w_{pq},\ \Gamma_{2p,t} = \begin{bmatrix} \mathscr{R}_t & \check{\tau}_{p,t}\mathscr{R}_t \\ \check{\tau}_{p,t}\mathscr{R}_t & \mathscr{T}_{p,t}\mathscr{R}_t \end{bmatrix}. \tag{19}$$

Then, $\mathfrak{X}_{p,t}$ *is bounded by* $\overline{\mathfrak{X}}_{p,t}$, *i.e.* $\mathfrak{X}_{p,t} \leqslant \overline{\mathfrak{X}}_{p,t}$.

Proof. It follows from (6) that

$$\begin{aligned}\mathfrak{X}_{p,t+1} = {} & \mathbb{E}\left\{\mathscr{X}_{p,t+1}\mathscr{X}_{p,t+1}^T\right\} \\ = {} & \mathbb{E}\Bigg\{\Bigg[\mathscr{A}_{1p,t}\mathscr{X}_{p,t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t \\ & + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} + v_{p,t}\mathscr{B}_{2p,t}\omega_t\Bigg] \\ & \times\Bigg[\mathscr{A}_{1p,t}\mathscr{X}_{p,t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t \\ & + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} + v_{p,t}\mathscr{B}_{2p,t}\omega_t\Bigg]^T\Bigg\} \\ = {} & \mathscr{A}_{1p,t}\mathbb{E}\left\{\mathscr{X}_{p,t}\mathscr{X}_{p,t}^T\right\}\mathscr{A}_{1p,t}^T + \mathbb{E}\Bigg\{\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} \\ & \times\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\} + \mathscr{B}_{1p,t}\mathbb{E}\left\{\tilde{\tau}_{2p,t}\omega_t\omega_t^T\tilde{\tau}_{2p,t}^T\right\}\mathscr{B}_{1p,t}^T \\ & + \mathbb{E}\left\{v_{p,t}^2\mathscr{A}_{2p,t}\mathscr{X}_{p,t}\mathscr{X}_{p,t}^T\mathscr{A}_{2p,t}^T\right\} + \mathbb{E}\Bigg\{v_{p,t}^2\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} \\ & \times\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\} + \mathbb{E}\left\{v_{p,t}^2\mathscr{B}_{2p,t}\omega_t\omega_t^T\mathscr{B}_{2p,t}^T\right\} \\ & + \mathfrak{A}_{p,t} + \mathfrak{A}_{p,t}^T + \mathfrak{B}_{p,t} + \mathfrak{B}_{p,t}^T + \mathfrak{I}_{p,t} + \mathfrak{I}_{p,t}^T + \mathfrak{K}_{p,t} + \mathfrak{K}_{p,t}^T \\ & + \mathfrak{L}_{p,t} + \mathfrak{L}_{p,t}^T + \mathfrak{M}_{p,t} + \mathfrak{M}_{p,t}^T + \mathfrak{N}_{p,t} + \mathfrak{N}_{p,t}^T \end{aligned} \tag{20}$$

where

$$
\begin{aligned}
\mathfrak{A}_{p,t} &\triangleq \mathbb{E}\left\{\mathscr{A}_{1p,t}\mathscr{X}_{p,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\right]^T\right\},\\
\mathfrak{B}_{p,t} &\triangleq \mathbb{E}\left\{v_{p,t}^2\mathscr{A}_{2p,t}\mathscr{X}_{p,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\right]^T\right\}.\\
\mathfrak{J}_{p,t} &\triangleq \mathbb{E}\left\{\mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t\left[\mathscr{A}_{1p,t}\mathscr{X}_{p,t}+\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\right]^T\right\},\\
\mathfrak{N}_{p,t} &\triangleq \mathbb{E}\left\{v_{p,t}^2\mathscr{B}_{2p,t}\omega_t\left[\mathscr{A}_{2p,t}\mathscr{X}_{p,t}+\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\right]^T\right\},\\
\mathfrak{K}_{p,t} &\triangleq \mathbb{E}\left\{\mathscr{A}_{1p,t}\mathscr{X}_{p,t}\left[v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t}+v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}+v_{p,t}\mathscr{B}_{2p,t}\omega_t\right]^T\right\},\\
\mathfrak{L}_{p,t} &\triangleq \mathbb{E}\left\{\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\left[v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t}+v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}+v_{p,t}\mathscr{B}_{2p,t}\omega_t\right]^T\right\},\\
\mathfrak{M}_{p,t} &\triangleq \mathbb{E}\left\{\mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t\left[v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t}+v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}+v_{p,t}\mathscr{B}_{2p,t}\omega_t\right]^T\right\}.
\end{aligned}
\tag{21}
$$

By using Assumption 1 and recalling the statistical characteristics of $v_{p,t}$ and ω_t, it is not difficult to obtain

$$\mathfrak{J}_{p,t}=0,\ \mathfrak{K}_{p,t}=0,\ \mathfrak{L}_{p,t}=0,\ \mathfrak{M}_{p,t}=0,\ \mathfrak{N}_{p,t}=0. \tag{22}$$

According to Lemma 1, one has

$$\mathfrak{A}_{p,t}+\mathfrak{A}_{p,t}^T \leqslant a_{p,t}\mathscr{A}_{1p,t}\mathbb{E}\left\{\mathscr{X}_{p,t}\mathscr{X}_{p,t}^T\right\}\mathscr{A}_{1p,t}^T+a_{p,t}^{-1}\mathbb{E}\left\{\sum_{q=1,q\neq p}^{Q} w_{pq}\ \times\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\right]^T\right\}, \tag{23}$$

and

$$\mathfrak{B}_{p,t}+\mathfrak{B}_{p,t}^T \leqslant b_{p,t}\mathbb{E}\left\{v_{p,t}^2\mathscr{A}_{2p,t}\mathscr{X}_{p,t}\mathscr{X}_{p,t}^T\mathscr{A}_{2p,t}^T\right\}+b_{p,t}^{-1}\mathbb{E}\left\{v_{p,t}^2\sum_{q=1,q\neq p}^{Q} w_{pq}\ \times\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\right]^T\right\}. \tag{24}$$

By making full use of Lemma 1 again, the following is achieved:

$$
\begin{aligned}
&\mathbb{E}\left\{\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\right]^T\right\}\\
&\quad\leqslant \frac{1}{2}\mathbb{E}\left\{\sum_{q=1,q\neq p}^{Q}\sum_{l=1,l\neq p}^{Q} w_{pq}w_{pl}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\left(\mathscr{X}_{q,t}\mathscr{X}_{q,t}^T+\mathscr{X}_{l,t}\mathscr{X}_{l,t}^T\right)\bar{\Pi}_{1p,t}^T\tilde{\tau}_{1p,t}^T\right\}\\
&\quad= \bar{w}_p\mathbb{E}\left\{\sum_{q=1,q\neq p}^{Q} w_{pq}\tilde{\tau}_{1p,t}\bar{\Pi}_{1p,t}\left(\mathscr{X}_{q,t}\mathscr{X}_{q,t}^T\right)\bar{\Pi}_{1p,t}^T\tilde{\tau}_{1p,t}^T\right\}\\
&\quad\leqslant \bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\mathfrak{X}_{q,t}\bar{\Pi}_{1p,t}^T\right)\mathbb{E}\left\{\tilde{\tau}_{1p,t}\tilde{\tau}_{1p,t}^T\right\}\\
&\quad= \bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\mathfrak{X}_{q,t}\bar{\Pi}_{1p,t}^T\right)\Gamma_{1p,t}.
\end{aligned}
\tag{25}
$$

Similarly, it can also be obtained that

$$\mathbb{E}\left\{ v_{p,t}^2 \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\left[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\right]^T\right\} \leqslant \tilde{v}_p\bar{w}_p \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathfrak{X}_{q,t}\bar{\Pi}_{2p,t}^T. \tag{26}$$

In addition, we have

$$\mathbb{E}\left\{\tilde{\tau}_{2p,t}\omega_t\omega_t^T\tilde{\tau}_{2p,t}^T\right\} = \begin{bmatrix} \mathbb{E}\{\omega_t\omega_t^T\} & \mathbb{E}\{\tau_{p,t}\omega_t\omega_t^T\} \\ \mathbb{E}\{\tau_{p,t}\omega_t\omega_t^T\} & \mathbb{E}\left\{\tau_{p,t}^2\omega_t\omega_t^T\right\}\end{bmatrix} = \Gamma_{2p,t}. \tag{27}$$

Substituting (22)–(27) into (20) yields

$$\begin{aligned}\mathfrak{X}_{p,t+1} \leqslant{}& (1+a_{p,t})\mathscr{A}_{1p,t}\mathfrak{X}_{p,t}\mathscr{A}_{1p,t}^T + \left(1+a_{p,t}^{-1}\right)\bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\mathfrak{X}_{q,t}\bar{\Pi}_{1p,t}^T\right)\Gamma_{1p,t}\\ &+\mathscr{B}_{1p,t}\Gamma_{2p,t}\mathscr{B}_{1p,t}^T + (1+b_{p,t})\tilde{v}_p\mathscr{A}_{2p,t}\mathfrak{X}_{p,t}\mathscr{A}_{2p,t}^T + \tilde{v}_p\mathscr{B}_{2p,t}\mathscr{R}_t\mathscr{B}_{2p,t}^T\\ &+\left(1+b_{p,t}^{-1}\right)\tilde{v}_p\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathfrak{X}_{q,t}\bar{\Pi}_{2p,t}^T.\end{aligned} \tag{28}$$

Then, $\mathfrak{X}_{p,t+1} \leqslant \overline{\mathfrak{X}}_{p,t+1}$ follows based on the mathematical induction. □

Remark 5. It is notable that, in this paper, a non-augmentation approach is adopted to handel the recursive filtering problem for CNs with hope to avoid the computation burden arising from augmentations. Within this framework, the information of the cross-covariances $\mathbb{E}\left\{\mathscr{X}_{q,t}\mathscr{X}_{l,t}^T\right\}$ in (25) is needed to finally obtain the upper bound of FEC. However, sometimes the computation of the cross-covariances is difficult, or even impossible in some practical applications. To overcome this limitation, the cross-covariances now have been bounded by their individual covariances as shown in (25), which are not required to be calculated any more.

Theorem 1. *Given positive scalars $c_{lp,t+1}, d_{lp,t+1}$ and $e_{lp,t+1}$ $(l = 1,2,3)$, suppose*

$$\begin{aligned}\overline{\mathscr{S}}_{p,t+1|t+1} ={}& (1+d_{3p,t+1})(1+e_{1p,t+1})(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})\overline{\mathscr{S}}_{p,t+1|t}(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})^T\\ &+(1+d_{3p,t+1})\left(1+e_{1p,t+1}^{-1}\right)\mathscr{K}_{p,t+1}tr\left[\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}_{p,t+1}^T\right]I\mathscr{K}_{p,t+1}^T\\ &+\left(1+d_{3p,t+1}^{-1}\right)(1+e_{2p,t+1})(1+e_{3p,t+1})n_y^2\overline{\mathscr{H}}_{p,t+1}\mathscr{K}_{p,t+1}\mathscr{K}_{p,t+1}^T\\ &+\left(1+d_{3p,t+1}^{-1}\right)(1+e_{2p,t+1})\left(1+e_{3p,t+1}^{-1}\right)\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}E_{p,t+1}\\ &\times\mathscr{Q}_{t+1}E_{p,t+1}^T\mathscr{K}_{p,t+1}^T+\left(1+d_{3p,t+1}^{-1}\right)\left(1+e_{2p,t+1}^{-1}\right)\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}\\ &\times tr\left[E_{p,t+1}\mathscr{Q}_{t+1}E_{p,t+1}^T\right]I\mathscr{K}_{p,t+1}^T\end{aligned} \tag{29}$$

holds with the initial condition $\overline{\mathscr{S}}_{p,0|0} = \left[\tilde{x}_p \quad \bar{v}_p\tilde{x}_p\bar{v}_p\tilde{x}_p \quad \bar{v}_p^2\tilde{x}_p + \tilde{v}_p\left(\tilde{x}_p+\bar{x}_p\bar{x}_p^T\right)\right]$, where $\overline{\mathscr{H}}_{p,0} = \delta_{p,0}^2$ and

$$\begin{aligned}\overline{\mathscr{S}}_{p,t+1|t} \triangleq{}& \varepsilon_{1p,t}\mathscr{A}_{1p,t}\overline{\mathscr{S}}_{p,t|t}\mathscr{A}_{1p,t}^T + \varepsilon_{2p,t}\bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{1p,t}^T\right)\\ &\times\Gamma_{3p,t}+\mathscr{B}_{1p,t}\Gamma_{2p,t}\mathscr{B}_{1p,t}^T+\varepsilon_{3p,t}\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\overline{\mathscr{S}}_{q,t|t}\\ &\times\bar{\Pi}_{1p,t}^T\bar{\tau}_{p,t}^T+(1+b_{p,t})\tilde{v}_p\mathscr{A}_{2p,t}\overline{\mathfrak{X}}_{p,t}\mathscr{A}_{2p,t}^T+\left(1+b_{p,t}^{-1}\right)\tilde{v}_p\bar{w}_p\\ &\times\sum_{p=1,p\neq q}^{Q} w_{qp}\bar{\Pi}_{2p,t}\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{2p,t}^T+\tilde{v}_p\mathscr{B}_{2p,t}\mathscr{R}_t\mathscr{B}_{2p,t}^T,\\ \overline{\mathscr{H}}_{p,t+1} \triangleq{}& (1+d_{1p,t})\gamma_p^2\overline{\mathscr{H}}_{p,t}+\epsilon_p^2\left(1+d_{1p,t}^{-1}\right)(1+d_{2p,t})tr\left[\bar{C}_{p,t}\overline{\mathscr{S}}_{p,t|t-1}\bar{C}_{p,t}^T\right]\\ &+\epsilon_p^2\left(1+d_{1p,t}^{-1}\right)\left(1+d_{2p,t}^{-1}\right)\mathscr{T}_{p,t}tr\left[E_{p,t}\mathscr{Q}_tE_{p,t}^T\right],\\ \Gamma_{3p,t} \triangleq{}& diag\left\{0,\left(\mathscr{T}_{p,t}-\check{\tau}_{p,t}^2\right)I\right\},\ \varepsilon_{1p,t}\triangleq 1+c_{1p,t}+c_{2p,t},\\ \varepsilon_{2p,t} \triangleq{}& 1+c_{1p,t}^{-1}+c_{3p,t},\ \varepsilon_{3p,t}\triangleq 1+c_{2p,t}^{-1}+c_{3p,t}^{-1}.\end{aligned} \tag{30}$$

Then, the FEC $\mathscr{S}_{p,t|t}$ is bounded by $\overline{\mathscr{S}}_{p,t|t}$, i.e. $\mathscr{S}_{p,t|t} \leqslant \overline{\mathscr{S}}_{p,t|t}$.

Proof. The mathematical induction method is used for this proof. When $t = 0$, one has $\mathscr{S}_{p,0|0} = \overline{\mathscr{S}}_{p,0|0}$ easily. Suppose that $\mathscr{S}_{p,t|t} \leqslant \overline{\mathscr{S}}_{p,t|t}$ is true. Then, we want to show $\mathscr{S}_{p,t+1|t+1} \leqslant \overline{\mathscr{S}}_{p,t+1|t+1}$.

First, denote by $\mathscr{S}_{p,t+1|t} \triangleq \mathbb{E}\left\{\widetilde{\mathscr{X}}_{p,t+1|t}\widetilde{\mathscr{X}}^T_{p,t+1|t}\right\}$ the prediction error covariance, which can be calculated with respect to (16) as follows:

$$
\begin{aligned}
\mathscr{S}_{p,t+1|t} = {} & \mathbb{E}\Bigg\{\Bigg[\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t \\
& + \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t} + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} \\
& + v_{p,t}\mathscr{B}_{2p,t}\omega_t\Bigg]\Bigg[\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t} + \sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} + \mathscr{B}_{1p,t} \\
& \times \tilde{\tau}_{2p,t}\omega_t + \sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t} + v_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t} \\
& + v_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t} + v_{p,t}\mathscr{B}_{2p,t}\omega_t\Bigg]^T\Bigg\} \\
= {} & \mathbb{E}\left\{\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t}\widetilde{\mathscr{X}}^T_{p,t|t}\mathscr{A}^T_{1p,t}\right\} + \mathbb{E}\Bigg\{\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} \\
& \times\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\} + \mathbb{E}\left\{\mathscr{B}_{1p,t}\tilde{\tau}_{2p,t}\omega_t\omega^T_t\tilde{\tau}^T_{2p,t}\mathscr{B}^T_{1p,t}\right\} \\
& + \mathbb{E}\Bigg\{\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg]^T\Bigg\} \\
& + \mathbb{E}\left\{v^2_{p,t}\mathscr{A}_{2p,t}\mathscr{X}_{p,t}\mathscr{X}^T_{p,t}\mathscr{A}^T_{2p,t}\right\} + \mathbb{E}\Bigg\{v^2_{p,t}\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq} \\
& \times\bar{\Pi}_{2p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\} + \mathbb{E}\left\{v^2_{p,t}\mathscr{B}_{2p,t}\omega_t\omega^T_t\mathscr{B}^T_{2p,t}\right\} + \mathfrak{C}_{p,t} + \mathfrak{C}^T_{p,t} + \mathfrak{D}_{p,t} + \mathfrak{D}^T_{p,t} + \mathfrak{G}_{p,t} + \mathfrak{G}^T_{p,t} + \mathfrak{B}_{p,t} + \mathfrak{B}^T_{p,t}
\end{aligned} \tag{31}
$$

where $\mathfrak{B}_{p,t}$ is defined in (21) and

$$
\begin{aligned}
\mathfrak{C}_{p,t} & \triangleq \mathbb{E}\Bigg\{\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t}\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\}, \\
\mathfrak{D}_{p,t} & \triangleq \mathbb{E}\Bigg\{\mathscr{A}_{1p,t}\widetilde{\mathscr{X}}_{p,t|t}\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg]^T\Bigg\}, \\
\mathfrak{G}_{p,t} & \triangleq \mathbb{E}\Bigg\{\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\Bigg]\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg]^T\Bigg\}.
\end{aligned}
$$

Repeating the similar derivation process as in (23)–(26), we obtain

$$\begin{aligned}
\mathscr{S}_{p,t+1|t} \leqslant\ & \varepsilon_{1p,t}\mathscr{A}_{1p,t}\mathscr{S}_{p,t|t}\mathscr{A}_{1p,t}^T + \varepsilon_{2p,t}\mathbb{E}\Bigg\{\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t} \\
& \times\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\vec{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{X}_{q,t}\Bigg]^T\Bigg\} + \mathscr{B}_{1p,t}\Gamma_{2p,t}\mathscr{B}_{1p,t}^T + \varepsilon_{3p,t} \\
& \times\mathbb{E}\Bigg\{\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg[\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\widetilde{\mathscr{X}}_{q,t|t}\Bigg]^T\Bigg\} \\
& + (1+b_{p,t})\tilde{v}_p\mathscr{A}_{2p,t}\overline{\mathfrak{X}}_{p,t}\mathscr{A}_{2p,t}^T + \left(1+b_{p,t}^{-1}\right)\tilde{v}_p\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t} \\
& \times\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{2p,t}^T + \tilde{v}_p\mathscr{B}_{2p,t}\mathscr{R}_t\mathscr{B}_{2p,t}^T \\
\leqslant\ & \varepsilon_{1p,t}\mathscr{A}_{1p,t}\mathscr{S}_{p,t|t}\mathscr{A}_{1p,t}^T + \varepsilon_{2p,t}\bar{w}_p\lambda_{max}\left(\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{1p,t}\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{1p,t}^T\right)\Gamma_{3p,t} \\
& + \varepsilon_{3p,t}\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\tau}_{p,t}\bar{\Pi}_{1p,t}\mathscr{S}_{q,t|t}\bar{\Pi}_{1p,t}^T\bar{\tau}_{p,t}^T + \mathscr{B}_{1p,t}\Gamma_{2p,t}\mathscr{B}_{1p,t}^T \\
& + (1+b_{p,t})\tilde{v}_p\mathscr{A}_{2p,t}\overline{\mathfrak{X}}_{p,t}\mathscr{A}_{2p,t}^T + \left(1+b_{p,t}^{-1}\right)\tilde{v}_p\bar{w}_p\sum_{q=1,q\neq p}^{Q} w_{pq}\bar{\Pi}_{2p,t} \\
& \times\overline{\mathfrak{X}}_{q,t}\bar{\Pi}_{2p,t}^T + \tilde{v}_p\mathscr{B}_{2p,t}\mathscr{R}_t\mathscr{B}_{2p,t}^T
\end{aligned} \tag{32}$$

where Lemma 3 is used and $\varepsilon_{lp,t}$ $(l = 1,2,3)$ are defined in (30).

Taking $\mathscr{S}_{p,t|t} \leqslant \overline{\mathscr{S}}_{p,t|t}$ into account, we have

$$\mathscr{S}_{p,t+1|t} \leqslant \overline{\mathscr{S}}_{p,t+1|t}. \tag{33}$$

Next, defining $\mathscr{H}_{p,t} \triangleq \mathbb{E}\left\{\delta_{p,t}^2\right\}$, it follows from (10) that

$$\begin{aligned}
\mathscr{H}_{p,t+1} =\ & \mathbb{E}\left\{\delta_{p,t+1}^2\right\} \\
=\ & \mathbb{E}\left\{\left(\gamma_p\delta_{p,t} + \epsilon_p\|\bar{y}_{p,t} - \bar{C}_{p,t}\widehat{\mathscr{X}}_{p,t|t-1}\|\right)^2\right\} \\
\leqslant\ & \gamma_p^2(1+d_{1p,t})\mathbb{E}\left\{\delta_{p,t}^2\right\} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)\mathbb{E}\left\{\|\bar{y}_{p,t} - \bar{C}_{p,t}\widehat{\mathscr{X}}_{p,t|t-1}\|^2\right\} \\
\leqslant\ & \gamma_p^2(1+d_{1p,t})\mathscr{H}_{p,t} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)(1+d_{2p,t})\mathbb{E}\left\{\widetilde{\mathscr{X}}_{p,t|t-1}^T\bar{C}_{p,t}^T\right. \\
& \left.\times\bar{C}_{p,t}\widetilde{\mathscr{X}}_{p,t|t-1}\right\} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)\left(1+d_{2p,t}^{-1}\right)\mathbb{E}\left\{\tau_{p,t}^2\upsilon_t^TE_{p,t}^TE_{p,t}\upsilon_t\right\} \\
=\ & \gamma_p^2(1+d_{1p,t})\mathscr{H}_{p,t} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)(1+d_{2p,t})\mathbb{E}\left\{tr\left[\bar{C}_{p,t}\widetilde{\mathscr{X}}_{p,t|t-1}\right.\right. \\
& \left.\left.\times\widetilde{\mathscr{X}}_{p,t|t-1}^T\bar{C}_{p,t}^T\right]\right\} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)\left(1+d_{2p,t}^{-1}\right)\mathscr{T}_{p,t}\mathbb{E}\left\{tr\left[E_{p,t}\upsilon_t\upsilon_t^TE_{p,t}^T\right]\right\} \\
=\ & \gamma_p^2(1+d_{1p,t})\mathscr{H}_{p,t} + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)(1+d_{2p,t})tr\left[\bar{C}_{p,t}\mathscr{S}_{p,t|t-1}\bar{C}_{p,t}^T\right] \\
& + \epsilon_p^2\left(1+d_{1p,t}^{-1}\right)\left(1+d_{2p,t}^{-1}\right)\mathscr{T}_{p,t}tr\left[E_{p,t}\mathscr{Q}_tE_{p,t}^T\right]
\end{aligned} \tag{34}$$

which, according to (33), indicates that

$$\mathscr{H}_{p,t+1} \leqslant \overline{\mathscr{H}}_{p,t+1}. \tag{35}$$

At last, let us now discuss the FEC $\mathscr{S}_{p,t+1|t+1}$. From (16), it is obvious that

$$\begin{aligned}\mathscr{S}_{p,t+1|t+1} &= \mathbb{E}\Big\{\widetilde{\mathscr{X}}_{p,t+1|t+1}\widetilde{\mathscr{X}}^T_{p,t+1|t+1}\Big\}\\ &= \mathbb{E}\Big\{\Big[\big(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\big)\widetilde{\mathscr{X}}_{p,t+1|t}+\mathscr{K}_{p,t+1}\Delta_{p,t+1}\bar{C}_{p,t+1}\\ &\quad\times\widetilde{\mathscr{X}}_{p,t+1|t}-\mathscr{K}_{p,t+1}\big(I-\Delta_{p,t+1}\big)\tau_{p,t+1}E_{p,t+1}\upsilon_{t+1}\\ &\quad-\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}\Big]\Big[\big(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\big)\widetilde{\mathscr{X}}_{p,t+1|t}\\ &\quad+\mathscr{K}_{p,t+1}\Delta_{p,t+1}\bar{C}_{p,t+1}\widetilde{\mathscr{X}}_{p,t+1|t}-\mathscr{K}_{p,t+1}\big(I-\Delta_{p,t+1}\big)\\ &\quad\times\tau_{p,t+1}E_{p,t+1}\upsilon_{t+1}-\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}{}^T\Big\}.\end{aligned}\tag{36}$$

By using Lemma 1, it is further obtained from (36) that

$$\begin{aligned}\mathscr{S}_{p,t+1|t+1} &\leqslant \big(1+d_{3p,t+1}\big)\big(1+e_{1p,t+1}\big)\big(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\big)\mathbb{E}\Big\{\widetilde{\mathscr{X}}_{p,t+1|t}\widetilde{\mathscr{X}}^T_{p,t+1|t}\Big\}\\ &\quad\times\big(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\big)^T+\big(1+d_{3p,t+1}\big)\Big(1+e^{-1}_{1p,t+1}\Big)\mathbb{E}\{\mathscr{K}_{p,t+1}\Delta_{p,t+1}\\ &\quad\times\bar{C}_{p,t+1}\widetilde{\mathscr{X}}_{p,t+1|t}\widetilde{\mathscr{X}}^T_{p,t+1|t}\bar{C}^T_{p,t+1}\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}+\Big(1+d^{-1}_{3p,t+1}\Big)\\ &\quad\times\big(1+e_{2p,t+1}\big)\big(1+e_{3p,t+1}\big)\mathbb{E}\Big\{\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}\Lambda^T_{p,t+1}\\ &\quad\times\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}+\Big(1+d^{-1}_{3p,t+1}\Big)\big(1+e_{2p,t+1}\big)\Big(1+e^{-1}_{3p,t+1}\Big)\\ &\quad\times\mathbb{E}\Big\{\tau^2_{p,t+1}\mathscr{K}_{p,t+1}E_{p,t+1}\upsilon_{t+1}\upsilon^T_{t+1}E^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}+\Big(1+d^{-1}_{3p,t+1}\Big)\\ &\quad\times\Big(1+e^{-1}_{2p,t+1}\Big)\mathbb{E}\Big\{\tau^2_{p,t+1}\mathscr{K}_{p,t+1}\Delta_{p,t+1}E_{p,t+1}\upsilon_{t+1}\upsilon^T_{t+1}E^T_{p,t+1}\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}.\end{aligned}\tag{37}$$

With the help of the properties of matrix trace, we have

$$\begin{aligned}&\mathbb{E}\Big\{\mathscr{K}_{p,t+1}\Delta_{p,t+1}\bar{C}_{p,t+1}\widetilde{\mathscr{X}}_{p,t+1|t}\widetilde{\mathscr{X}}^T_{p,t+1|t}\bar{C}^T_{p,t+1}\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}\\ &\quad\leqslant\mathscr{K}_{p,t+1}\mathbb{E}\Big\{tr\Big[\Delta_{p,t+1}\bar{C}_{p,t+1}\widetilde{\mathscr{X}}_{p,t+1|t}\widetilde{\mathscr{X}}^T_{p,t+1|t}\bar{C}^T_{p,t+1}\Delta^T_{p,t+1}\Big]I\Big\}\mathscr{K}^T_{p,t+1}\\ &\quad\leqslant\mathscr{K}_{p,t+1}tr\Big[\bar{C}_{p,t+1}\mathscr{S}_{p,t+1|t}\bar{C}^T_{p,t+1}\Big]I\mathscr{K}^T_{p,t+1},\\ &\mathbb{E}\Big\{\tau^2_{p,t+1}\mathscr{K}_{p,t+1}\Delta_{p,t+1}E_{p,t+1}\upsilon_{t+1}\upsilon^T_{t+1}E^T_{p,t+1}\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}\\ &\quad\leqslant\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}\mathbb{E}\Big\{tr\Big[\Delta_{p,t+1}E_{p,t+1}\upsilon_{t+1}\upsilon^T_{t+1}E^T_{p,t+1}\Delta^T_{p,t+1}\Big]I\Big\}\mathscr{K}^T_{p,t+1}\\ &\quad\leqslant\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}tr\Big[E_{p,t+1}\mathscr{Q}_{t+1}E^T_{p,t+1}\Big]I\mathscr{K}^T_{p,t+1}\end{aligned}\tag{38}$$

and

$$\begin{aligned}&\mathbb{E}\Big\{\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}\Lambda^T_{p,t+1}\Delta^T_{p,t+1}\mathscr{K}^T_{p,t+1}\Big\}\\ &\leqslant\mathscr{K}_{p,t+1}\mathbb{E}\Big\{tr\Big[\Delta_{p,t+1}\Lambda_{p,t+1}\Lambda^T_{p,t+1}\Delta^T_{p,t+1}\Big]I\Big\}\mathscr{K}^T_{p,t+1}\\ &\leqslant\mathscr{K}_{p,t+1}tr\Big[\mathbb{E}\Big\{\Lambda_{p,t+1}\Lambda^T_{p,t+1}\Big\}\Big]I\mathscr{K}^T_{p,t+1}.\end{aligned}\tag{39}$$

Bearing in mind the definition of $\Lambda_{p,t+1}$, one has

$$\mathbb{E}\Big\{\Lambda_{p,t+1}\Lambda^T_{p,t+1}\Big\}\leqslant\mathbb{E}\Big\{tr\Big[\Lambda^T_{p,t+1}\Lambda_{p,t+1}\Big]\Big\}I=n_y\mathbb{E}\Big\{\delta^2_{p,t+1}\Big\}I\leqslant n_y\overline{\mathscr{H}}_{p,t+1}I\tag{40}$$

which further guarantees that

$$\mathbb{E}\left\{\mathscr{K}_{p,t+1}\Delta_{p,t+1}\Lambda_{p,t+1}\Lambda^{T}_{p,t+1}\Delta^{T}_{p,t+1}\mathscr{K}^{T}_{p,t+1}\right\} \leqslant n_y^2\overline{\mathscr{H}}_{p,t+1}\mathscr{K}_{p,t+1}\mathscr{K}^{T}_{p,t+1} \tag{41}$$

Substituting (38) and (41) into (37) yields

$$\begin{aligned}\mathscr{S}_{p,t+1|t+1} \leqslant\ & (1+d_{3p,t+1})(1+e_{1p,t+1})(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})\mathscr{S}_{p,t+1|t} \\ & \times (I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})^{T} + (1+d_{3p,t+1})\left(1+e^{-1}_{1p,t+1}\right) \\ & \times \mathscr{K}_{p,t+1}tr\left[\bar{C}_{p,t+1}\mathscr{S}_{p,t+1|t}\bar{C}^{T}_{p,t+1}\right]I\mathscr{K}^{T}_{p,t+1} + \left(1+d^{-1}_{3p,t+1}\right) \\ & \times (1+e_{2p,t+1})(1+e_{3p,t+1})n_y^2\overline{\mathscr{H}}_{p,t+1}\mathscr{K}_{p,t+1}\mathscr{K}^{T}_{p,t+1} \\ & + \left(1+d^{-1}_{3p,t+1}\right)(1+e_{2p,t+1})\left(1+e^{-1}_{3p,t+1}\right)\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1} \\ & \times E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1}\mathscr{K}^{T}_{p,t+1} + \left(1+d^{-1}_{3p,t+1}\right)\left(1+e^{-1}_{2p,t+1}\right) \\ & \times \mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}tr\left[E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1}\right]I\mathscr{K}^{T}_{p,t+1}.\end{aligned} \tag{42}$$

According to (27), it is clear that

$$\mathscr{S}_{p,t+1|t+1} \leqslant \overline{\mathscr{S}}_{p,t+1|t+1}. \tag{43}$$

The proof is complete now. □

In the following theorem, the gain matrix of the outlier-resistant filter is determined.

Theorem 2. *The upper bound $\overline{\mathscr{S}}_{p,t+1|t+1}$ in Theorem 1 achieves minimum with the following filter gain:*

$$\mathscr{K}_{p,t+1} = \Upsilon_{1p,t+1}\Upsilon^{-1}_{2p,t+1} \tag{44}$$

where

$$\begin{aligned}\Upsilon_{1p,t+1} \triangleq\ & (1+d_{3p,t+1})(1+e_{1p,t+1})\overline{\mathscr{S}}_{p,t+1|t}\bar{C}^{T}_{p,t+1}, \\ \Upsilon_{2p,t+1} \triangleq\ & (1+d_{3p,t+1})(1+e_{1p,t+1})\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}^{T}_{p,t+1} + (1+d_{3p,t+1}) \\ & \times \left(1+e^{-1}_{1p,t+1}\right)tr\left[\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}^{T}_{p,t+1}\right]I + \left(1+d^{-1}_{3p,t+1}\right) \\ & \times (1+e_{2p,t+1})(1+e_{3p,t+1})n_y^2\overline{\mathscr{H}}_{p,t+1}I + \left(1+d^{-1}_{3p,t+1}\right) \\ & \times (1+e_{2p,t+1})\left(1+e^{-1}_{3p,t+1}\right)\mathscr{T}_{p,t+1}E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1} \\ & + \left(1+d^{-1}_{3p,t+1}\right)tr\left[E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1}\right]I.\end{aligned} \tag{45}$$

Proof. According to (27), we derive that

$$\begin{aligned}\mathrm{tr}\left[\overline{\mathscr{S}}_{p,t+1|t+1}\right] =\ & (1+d_{3p,t+1})(1+e_{1p,t+1})\mathrm{tr}\left[(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})\overline{\mathscr{S}}_{p,t+1|t}\right. \\ & \left.\times(I-\mathscr{K}_{p,t+1}\bar{C}_{p,t+1})^{T}\right] + (1+d_{3p,t+1})\left(1+e^{-1}_{1p,t+1}\right) \\ & \times \mathrm{tr}\left[\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}^{T}_{p,t+1}\right]\mathrm{tr}\left[\mathscr{K}_{p,t+1}\mathscr{K}^{T}_{p,t+1}\right] + \left(1+d^{-1}_{3p,t+1}\right) \\ & \times (1+e_{2p,t+1})(1+e_{3p,t+1})n_y^2\overline{\mathscr{H}}_{p,t+1}\mathrm{tr}\left[\mathscr{K}_{p,t+1}\mathscr{K}^{T}_{p,t+1}\right] \\ & + \left(1+d^{-1}_{3p,t+1}\right)(1+e_{2p,t+1})\left(1+e^{-1}_{3p,t+1}\right)\mathscr{T}_{p,t+1}\mathrm{tr}\left[\mathscr{K}_{p,t+1}\right. \\ & \left.\times E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1}\mathscr{K}^{T}_{p,t+1}\right] + \left(1+d^{-1}_{3p,t+1}\right)\left(1+e^{-1}_{2p,t+1}\right) \\ & \times \mathscr{T}_{p,t+1}\mathrm{tr}\left[E_{p,t+1}\mathscr{Q}_{t+1}E^{T}_{p,t+1}\right]\mathrm{tr}\left[\mathscr{K}_{p,t+1}\mathscr{K}^{T}_{p,t+1}\right].\end{aligned} \tag{46}$$

Moreover,

$$\begin{aligned}\frac{\partial \mathrm{tr}\left[\overline{\mathscr{S}}_{p,t+1|t+1}\right]}{\partial \mathscr{K}_{p,t+1}} = & -2(1+d_{3p,t+1})(1+e_{1p,t+1})\overline{\mathscr{S}}_{p,t+1|t}\bar{C}_{p,t+1}^{T} + 2(1+d_{3p,t+1}) \\ & \times (1+e_{1p,t+1})\mathscr{K}_{p,t+1}\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}_{p,t+1}^{T} + 2(1+d_{3p,t+1}) \\ & \times \left(1+e_{1p,t+1}^{-1}\right)\mathscr{K}_{p,t+1}\mathrm{tr}\left[\bar{C}_{p,t+1}\overline{\mathscr{S}}_{p,t+1|t}\bar{C}_{p,t+1}^{T}\right] + 2\left(1+d_{3p,t+1}^{-1}\right) \\ & \times (1+e_{2p,t+1})(1+e_{3p,t+1})n_{y}^{2}\overline{\mathscr{H}}_{p,t+1}\mathscr{K}_{p,t+1} + 2\left(1+d_{3p,t+1}^{-1}\right) \\ & \times (1+e_{2p,t+1})\left(1+e_{3p,t+1}^{-1}\right)\mathscr{T}_{p,t+1}\mathscr{K}_{p,t+1}E_{p,t+1}\mathscr{Q}_{t+1}E_{p,t+1}^{T} \\ & + 2\left(1+d_{3p,t+1}^{-1}\right)\left(1+e_{2p,t+1}^{-1}\right)\mathscr{T}_{p,t+1}\mathrm{tr}\left[E_{p,t+1}\mathscr{Q}_{t+1}E_{p,t+1}^{T}\right]\mathscr{K}_{p,t+1}.\end{aligned} \tag{47}$$

For the purpose of minimizing $\mathrm{tr}\left[\overline{\mathscr{S}}_{p,t+1|t+1}\right]$, we let

$$\frac{\partial \mathrm{tr}\left[\overline{\mathscr{S}}_{p,t+1|t+1}\right]}{\partial \mathscr{K}_{p,t+1}} = 0 \tag{48}$$

which results in

$$\mathscr{K}_{p,t+1} = \Upsilon_{1p,t+1}\Upsilon_{2p,t+1}^{-1}, \tag{49}$$

and the proof is therefore complete. □

Till now, the outlier-resistant recursive filtering problem has been solved. The design procedure of the recursive filtering strategy can be summarized in the following algorithm (Algorithm 1).

Algorithm 1: outlier-resistant filter

Step 1. Let $t = 0$ and the maximum iterative step be $t_{\max}$. For node p, given the initial values $\bar{x}_p$, $\tilde{x}_p$, $\delta_{p,0}$, $\bar{v}_p$, $\tilde{v}_p$ and then obtain $\overline{\mathscr{S}}_{p,0|0}$ as well as $\bar{\mathfrak{x}}_{p,0}$ based on these initial values;
Step 2. Compute the desired filter gain $\mathscr{K}_{p,t+1}$ from (44);
Step 3. According to the filter structure (8), calculate the estimate $\widehat{\mathscr{X}}_{p,t+1|t+1}$ and then obtain the estimate of network state $x_{p,t+1}$ with $\hat{x}_{p,t+1|t+1} = [I \quad 0]\widehat{\mathscr{X}}_{p,t+1|t+1}$.
Step 4. Solve $\overline{\mathscr{S}}_{p,t+1|t+1}$ in terms of the recursion (29) and set $t = t + 1$;
Step 5. If $t < t_{\max}$, go back to Step 2, else Stop.

Remark 6. The outlier-resistant filter design issue has been dealt with in Theorems 1,2 for CNs with TcFCs. To be specific, a sufficient condition has been proposed in Theorem 1 to guarantee FEC bounds exist by means of mathematical induction. Subsequently, in Theorem 2, we have calculated the gain by recursion such that minimal upper bound is achieved. Obviously, it is seen from Theorems 1,2 that our main results reflect all the information about the network parameters as well as the dynamics of both fading coefficient and saturation level sufficiently. Furthermore, in order to facilitate further implementation, the proposed outlier-resistant filter design algorithm has been summarized in Algorithm 1 regarding the addressed recursive filtering issue. Bearing in mind the dimensions of related variables, at each iteration, it is not difficult to obtain that the time complexity of this algorithm is $O(n_x^3)$ and the space complexity of this algorithm is $O(n_x^2)$.

Remark 7. The outlier-resistant filter design issue for CNs has gained much research attention and therefore there have been a large number of effective filtering methods available. Compared with existing studies, the filter possesses distinctive novelties: 1) this design issue is new for CNs with TcFCs and outliers; 2) a saturation structure is introduced to against possible outliers where certain saturation level can be regulated dynamically according to the innovations; and 3) outlier-resistant filtering is proposed to parameterize filter gains recursively.

4. Numerical example

In this section, we present two numerical examples for the CN (1) with the filter (8).

4.1. Example 1

In this example, we consider a CN (1) having three nodes with the relevant parameters set as follows:

$$A_{1,t} = A_{2,t} = A_{3,t} = \begin{bmatrix} 0.25 & 0.2 \\ 0.2 & 0.3 - 0.01\sin(2t) \end{bmatrix}, B_{1,t} = \begin{bmatrix} 0.4 \\ 0.4 + 0.01\sin(2t) \end{bmatrix},$$
$$B_{2,t} = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix}, B_{3,t} = \begin{bmatrix} 0.5 + 0.01\cos(2t) \\ 0.3 \end{bmatrix}, \mathscr{W} = \begin{bmatrix} -0.3 & 0.15 & 0.15 \\ 0.15 & -0.3 & 0.15 \\ 0.15 & 0.15 & -0.3 \end{bmatrix}, \Pi = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

The matrices in the measurement output (2) are given by

$$C_{1,t} = [\,1.2 \quad 0.8 + 0.01\cos(2t)\,], C_{2,t} = [\,0.8 + 0.01\sin(2t) \quad 1\,],$$
$$C_{3,t} = [\,1 + 0.01\cos(2t) \quad 0.9 + 0.01\cos(2t)\,],$$
$$E_{1,t} = 0.5, E_{2,t} = 0.3, E_{3,t} = 0.4.$$

Considering the TcFCs (3)–(4) and the adaptive saturation level (10), some corresponding parameters are chosen as $\lambda_{p,t} = 0.99$ $(p = 1,2,3)$, $\bar{\nu}_p = 0.8, \tilde{\nu}_p = 0.1, \gamma_p = 0.95, \epsilon_p = 0.01$ and $\delta_{p,0} = 0.5$. Moreover, for the covariances of noises, we set $\mathscr{R}_t = 0.5$ and $\mathscr{Q}_t = 0.5$. The initial value $x_{p,0}$ is assumed to has mean $\bar{x}_p = 0$ and covariance $\tilde{x}_p = 0.5$. With these given parameters, the desired filter parameters and the upper bound of FEC for each network node can be calculated recursively by resorting to Algorithm 1.

For node p $(p = 1,2,3)$, let $\mathrm{MSE}_{p,t}$ be defined as

$$\mathrm{MSE}_{p,t} = \frac{1}{S}\sum_{t=1}^{S}\sum_{r=1}^{4}\left(\mathscr{X}^r_{p,t} - \widehat{\mathscr{X}}^r_{p,t|t}\right)^2$$

where S is independent experiments, $\mathscr{X}^r_{p,t}$ and $\widehat{\mathscr{X}}^r_{p,t|t}$ are the rth elements of state $\mathscr{X}_{p,t}$ and estimate $\widehat{\mathscr{X}}_{p,t|t}$, respectively. The simulation runs for $S = 300$ times and the results are shown in Figs. 1–3, where Figs. 1,2 display node trajectories and estimates in CNs. Fig. 3 shows the $\mathrm{MSE}_{p,t}$ and the trace of their upper bounds. It is clear from Fig. 3 that $\mathrm{MSE}_{p,t}$ is always smaller than the trace of their upper bounds, which conforms to the theoretical analysis.

On one hand, in order to verify that our proposed filtering method could attenuate the side effects of possible outliers, we introduce zero-mean Gaussian variables with covariances 1000 to characterize outliers, which take place periodically with a period of 3 instants. For comparison, consider three cases including adaptive saturation level (i.e. the mechanism considered in this paper), fixed saturation level and infinite saturation level (i.e. the case without saturation) on the recursive filtering problem for CNs described above. The comparative results of the $\mathrm{MSE}_{p,t}$ with different saturation levels are plotted in Fig. 4, where estimate errors with adaptive saturation level are almost smaller than the ones of other two cases, which implies that our proposed algorithm with adaptive saturation level is indeed effective to attenuate the impacts of possible outliers, hence improving the estimation performance largely.

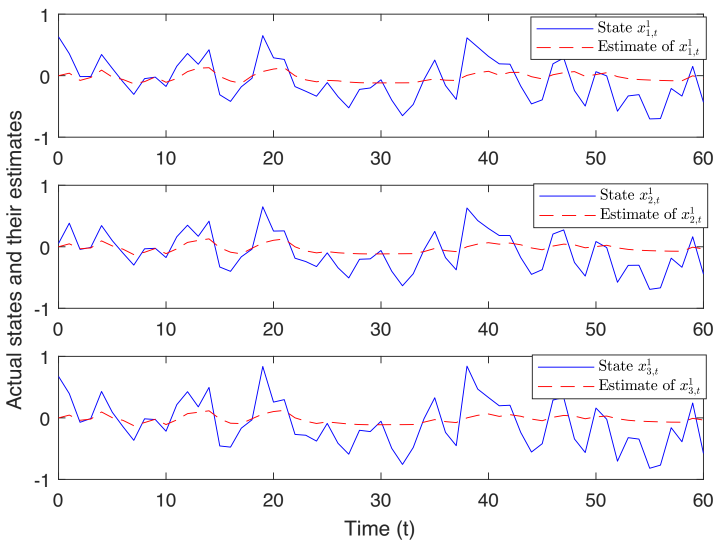

Fig. 1. States $x^1_{p,t}$ $(p = 1,2,3)$ and their corresponding estimates.

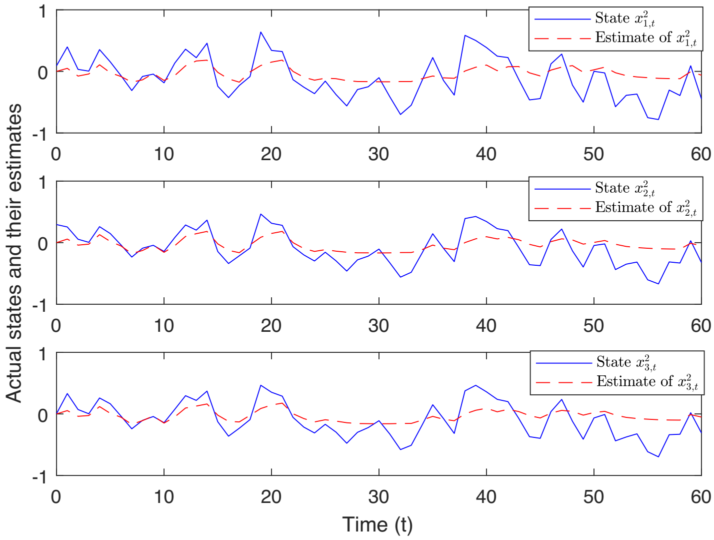

Fig. 2. States $x^2_{p,t}$ $(p = 1, 2, 3)$ and their corresponding estimates.

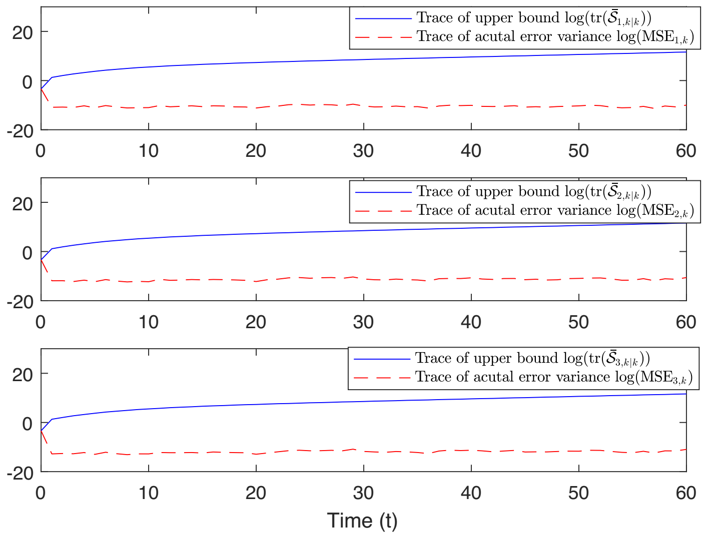

Fig. 3. For nodes 1, 2 and 3, the mean-square errors and the trace of their corresponding upper bounds.

On the other hand, in order to further verify that our developed filtering algorithm is robust in dealing with model-mismatch and unknown disturbances, we consider that the underlying CN is subject to both parameter uncertainties and unknown disturbances in this simulation. The dynamics of node p $(p = 1, 2, 3)$ can then be rewritten as

$$x_{p,t+1} = (A_{p,t} + \Delta A_{p,t})x_{p,t} + \sum_{q=1}^{Q} w_{pq}\Pi x_{q,t} + d_{p,t} + B_{p,t}\omega_t,$$

where the parameter uncertainty $\Delta A_{p,t}$ is set as $\begin{bmatrix} 0.01 & -0.01 + 0.01\sin(2t) \\ 0 & 0.02 \end{bmatrix}$ and unknown disturbance $d_{p,t}$ is chosen as a zero-mean Gaussian variable with covariance 0.01. By operating Algorithm 1, the simulation results are shown in Figs. 5,6, which indicates that the robustness of the designed recursive filter can be guaranteed.

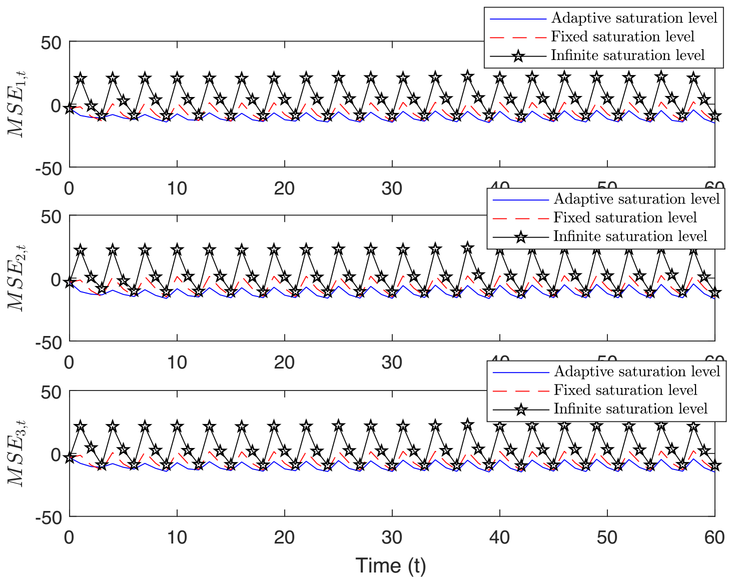

Fig. 4. The mean-square errors with different saturation levels for nodes 1, 2 and 3.

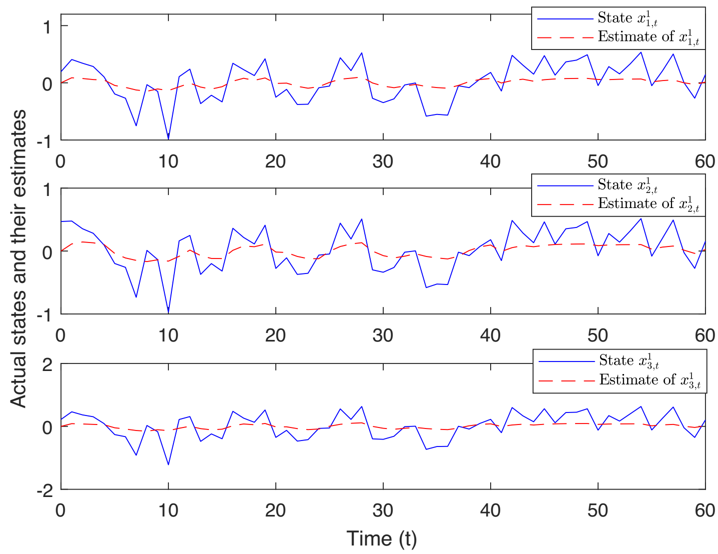

Fig. 5. States $x^1_{p,t}$ $(p = 1, 2, 3)$ and their corresponding estimates.

4.2. Example 2

In this example, in order to verify the practicability of the developed recursive filter design approach, we consider a class of complex networks constructed with 5 coupled RLC circuits [22]. The dynamics of node p can be described by

$$\dot{\phi}_{L_p} = -\frac{1}{C_p} q_{C_p} - \frac{R_p}{L_p}\phi_{L_p} + u_p$$
$$\dot{q}_{C_p} = \frac{1}{L_p}\phi_{L_p}, \quad p = 1, 2, \ldots, 5$$

where ϕ_{L_p} is the flux in the inductance, q_{C_p} stands for the charge in the capacitor and u_p is the control input. R_p, L_p and C_p denote the resistance, the inductance and the capacitor, respectively.

For node p, letting $x_p \triangleq \begin{bmatrix} x_p^1 & x_p^2 \end{bmatrix}^T$ with $x_p^1 \triangleq \phi_{L_p}$ and $x_p^2 \triangleq q_{C_p}$, the state-space model of RLC circuit is obtained as

$$\dot{x}_p = \bar{A}_p x_p + \bar{F}_p u_p \quad p = 1, 2, \ldots, 5$$

where $\bar{A}_p \triangleq \begin{bmatrix} -\frac{R_p}{L_p} & -\frac{1}{C_p} \\ \frac{1}{L_p} & 0 \end{bmatrix}$ and $\bar{F}_p \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$.

Discretizing the above the state-space model with period h, we have

$$x_{p,t+1} = A_p x_{p,t} + F_p u_{p,t} \quad p = 1, 2, \ldots, 5$$

with $A_p \triangleq e^{\bar{A}_p h}$ and $F_p = \int_0^h e^{\bar{A}_p s} ds \bar{F}$.

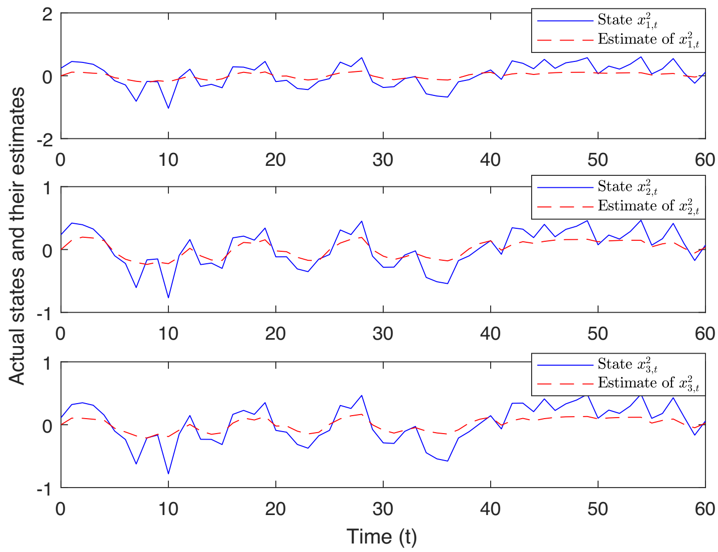

Fig. 6. States $x^2_{p,t}$ $(p = 1, 2, 3)$ and their corresponding estimates.

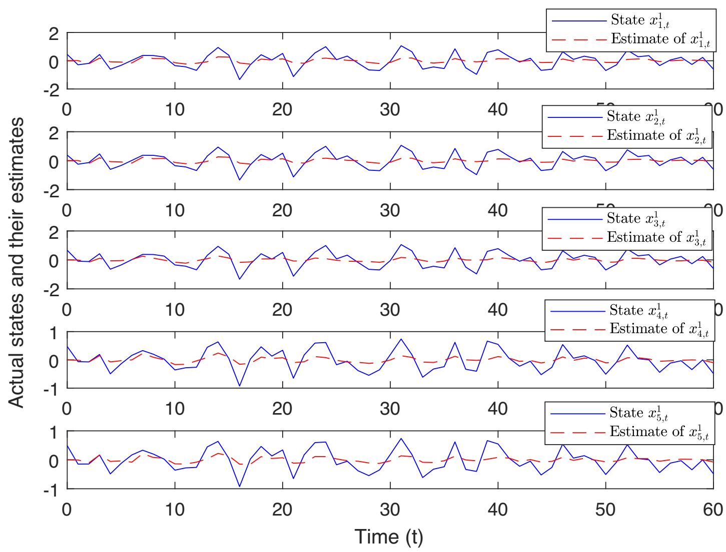

Fig. 7. States $x^1_{p,t}$ $(p = 1, 2, 3, 4, 5)$ and their corresponding estimates.

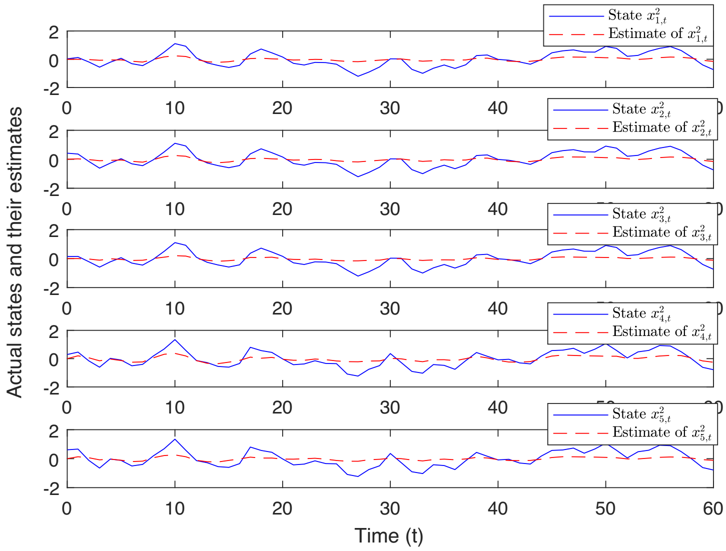

Fig. 8. States $x^2_{p,t}$ $(p = 1, 2, 3, 4, 5)$ and their corresponding estimates.

By designing the controller as $u_{p,t} = \sum_{q=1}^{Q} w_{pq} x^1_{q,t}$ and considering the external noise, the local dynamics of the circuit network can then be obtained as

$$x_{p,t+1} = A_{p,t} x_{p,t} + \sum_{q=1}^{Q} w_{pq} \Pi x_{q,t} + B_{p,t}\omega_t, \quad p = 1, 2, \ldots, 5.$$

where $A_{p,t} \triangleq A_p, \Pi \triangleq \int_0^h e^{\bar{A}_p s} ds \tilde{F}$ and $\tilde{F} \triangleq \mathrm{diag}\{I, 0\}$.

By selecting $R_p = 1\Omega, L_p = 0.5H, C_p = 0.5F$ and $h = 0.5s$, one has

$$A_{1,t} = A_{2,t} = A_{3,t} = A_{4,t} = A_{5,t} = \begin{bmatrix} 0.1262 & -0.5335 \\ 0.5335 & 0.6597 \end{bmatrix}, \quad \Pi = \mathrm{diag}\{0.26, 0\},$$

and the other parameters are chosen as

$$B_{1,t} = B_{2,t} = B_{3,t} = \begin{bmatrix} 0.6 \\ 0.3 + 0.01\sin(2t) \end{bmatrix}, B_{4,t} = B_{5,t} = \begin{bmatrix} 0.3 \\ 0.5 - 0.01\cos(2t) \end{bmatrix},$$
$$\mathscr{W} = \begin{bmatrix} -0.4 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & -0.4 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & -0.4 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & -0.4 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & -0.4 \end{bmatrix}, C_{1,t} = C_{2,t} = [1 \quad 0.9 + 0.01\cos(2t)],$$

$$C_{3,t} = C_{4,t} = [0.8 + 0.01\sin(2t) \quad 1.2], C_{5,t} = [1 \quad 1.2], E_{1,t} = 0.5,$$
$$E_{2,t} = E_{3,t} = 0.4, E_{4,t} = E_{5,t} = 0.6.$$

For each node p, the rest parameters are chosen as the same as those in Example 1. The corresponding results are shown in Figs. 7,8, from which we can see that the network state can be well estimated by employing the proposed filter design method.

5. Conclusion

In this paper, outlier-resistant filtering has been tackled with for CNs with TcFCs. Each sensor communicates with its corresponding filter through a TcFC whose channel coefficient has been governed by a dynamical process. To ensure resilience against possible outliers, a saturation structure has been introduced to restrict innovations polluted by outliers. An augmented model is constructed to combine channel and network dynamic evolutions. Then, with the inductive method, the FEC bound has been both obtained and minimized with suitable gains. Two examples are given for effectiveness validation. Future directions would be to on extension to sophisticated network phenomena [3,13,6,25,10].

CRediT authorship contribution statement

Qi Li: Conceptualization, Methodology, Software, Writing – original draft. **Zidong Wang:** Supervision, Conceptualization, Methodology, Writing – review & editing. **Hongli Dong:** Methodology, Writing – review & editing. **Weiguo Sheng:** Methodology, Software, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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