Information Sciences 630 (2023) 74–81

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Synchronization of machine learning oscillators in complex networks

Tongfeng Weng [a,*], Xiaolu Chen [b], Zhuoming Ren [a], Huijie Yang [b], Jie Zhang [c], Michael Small [d,e,*]

[a] Institute of Information Economy and Alibaba Business College, Hangzhou Normal University, Hangzhou 311121, PR China
[b] Business School, University of Shanghai for Science and Technology, Shanghai 200093, PR China
[c] Institute of Science and Technology for Brain-Inspired Intelligence, Fudan University, Shanghai 200433, China
[d] Complex Systems Group, Department of Mathematics and Statistics, The University of Western Australia, Crawley, WA 6009, Australia
[e] Mineral Resources, CSIRO, Kensington, WA 6151, Australia

A R T I C L E I N F O

Keywords:
Reservoir computing approach
Complex networks
Synchronization
Bifurcation

A B S T R A C T

We study synchronization phenomena in complex networks in terms of machine learning oscillators without conventional dynamical equations. Specifically, we adopt an effective machine learning technique known as reservoir computing for modeling dynamical systems of interest. By constructing a coupled configuration, we show that a collection of coupled reservoir oscillators are in identical synchrony over a wider window of coupling strengths. We find that the geometrical and dynamical properties of synchronous orbits are in excellent agreement with that of the learned dynamical system. Remarkably, through this synchronization scheme, we successfully recover an almost identical bifurcation behavior of an observed system via merely its chaotic dynamics. Our work provides an alternative framework for studying synchronization phenomena in nature when only observed data are available.

1. Introduction

Synchronization phenomena are ubiquitous in natural and man-made systems ranging from synchronous flashing of fireflies [1], rhythmic contraction of cardiac cells [2], to electric power grids [3]. It has been gradually recognized that a majority of these interesting phenomena can be collectively modeling and characterizing via synchronization in complex networks [4,5]. Extensive work has been done to study synchronization in various types of complex networks ranging from static networks [6,7] over time-varying networks [8] to multilayer networks [9]. These studies have revealed a great variety of intriguing collective behaviors, for example, chimeras [10], explosive synchronization [9], and cluster synchronization [11]. These findings have the potential application in social, physiological, and engineering systems, such as epileptic seizures in the brain [12] and bridge oscillations [13].

However, the previous studies of synchronization on complex networks have been limited to dynamical oscillators — that is the oscillator has an analytical equation, such as the Kuramoto model [5], Pulse-coupled models [14], and coupled maps [4]. While this restriction is extremely convenient for analytical tractability, it is often far from realistic. In fact, we frequently encounter synchronization phenomena in our life, for example, rhythmic contraction of cardiac cells [2] and applause in the theatre, whose

* Corresponding authors.
E-mail addresses: wtongfeng2006@163.com (T. Weng), michael.small@uwa.edu.au (M. Small).

https://doi.org/10.1016/j.ins.2023.02.028
Received 21 September 2021; Received in revised form 1 November 2022; Accepted 9 February 2023

Available online 13 February 2023

dynamical equations of the oscillators are unknown. While it is clear that studying synchronization on networks in terms of the oscillators without analytical equations will bring new insights to synchronization, a general framework has not yet been constructed.

Fortunately, the past few years have witnessed dramatic advances in the realm of artificial intelligence, from facial recognition [15] to playing go [16]. In particular, a celebrated machine learning technique known as reservoir computing approach has furnished us with an appealing way to model dynamical systems [17–19]. A growing number of studies have demonstrated that this technique delivers effective short-term prediction [18], and also resembles the long-term behavior of the dynamical system under consideration [20,21]. Here, we show that the reservoir systems can be used as the oscillators on networks rather than the conventional analytical equations, for example, the Kuramoto oscillator [5]. Specifically, we provide a coupled scheme for achieving synchronization of the reservoir oscillators in complex networks. We further show that correlation dimension and recurrence time of the synchronous orbit are identical with that of an original dynamical system. Interestingly, we find an intriguing bifurcation phenomenon in the coherent behaviors of the reservoir oscillators. Our work opens a new path to study synchronization in complex networks via the machine learning technique.

2. Model description

2.1. Synchronization of the reservoir oscillators

We begin by considering a network of N-coupled reservoir oscillators for which each reservoir oscillator is a reservoir computer. The coupling dynamical state of each reservoir oscillator is given by:

$$\mathbf{s}_i(t) = \rho\mathbf{u}_i(t) + \frac{1-\rho}{k_i}\sum_{j=1}^{N} a_{ij}\mathbf{u}_j(t), \tag{1}$$

where $\mathbf{u}_i(t)$ is the dynamical state of the ith reservoir oscillator at time t, ρ is the overall coupling strength lying in the range (0,1), a_{ij} is an element of the network's adjacency matrix, and $k_i = \sum_j a_{ij}$ is the degree of node i. The evolution of the ith reservoir oscillator is governed by:

$$\mathbf{r}_i(t+1) = (1-\alpha)\mathbf{r}_i(t) + \alpha\tanh\left(\tilde{\mathbf{A}}\mathbf{r}_i(t) + \mathbf{W}_{in}\begin{pmatrix} b_{in} \\ \mathbf{s}_i(t)\end{pmatrix}\right), \tag{2}$$

where $\mathbf{r}_i(t+1)$ is the reservoir state of its associated reservoir computer. For a learned dynamical system of interest, $\tilde{\mathbf{A}}$, $\mathbf{W}_{in}$, b_{in}, b_{out}, and α are the reservoir parameters usually given in advance before training [22,23]. Specifically, $\tilde{\mathbf{A}}$ is the adjacency matrix of a sparse random network. The elements in matrix $\mathbf{W}_{in}$ are drawn from a uniform distribution [-1,1], while the parameter α is a "leakage" rate lying in the range $[0,1]$. Normally, we set the bias $b_{in} = 1$ for convenience. Consequently, the output dynamical state $\mathbf{u}_i(t+1)$ of the ith reservoir oscillator is such that:

$$\mathbf{u}_i(t+1) = \mathbf{W}_{out}\begin{pmatrix} b_{out} \\ \mathbf{s}_i(t) \\ \mathbf{r}_i(t+1)\end{pmatrix}, \tag{3}$$

where the bias $b_{out} = 1$. The output weighted matrix $\mathbf{W}_{out}$ is the sole fitting parameter to be determined in the training phase. This parameter can be analytically calculated in terms of observed data [24]. After the successful train, we use the output dynamical state $\mathbf{u}_i(t+1)$ feedback to approximate the coupling dynamical input state $\mathbf{s}_i(t+1)$ and then the reservoir oscillator can run autonomously. Here, we are interested in what will happen among these coupled reservoir oscillators in the course of time evolution.

2.2. Synchronization of the reservoir oscillators upon learning the Hénon map

We first consider each reservoir oscillator modeling the Hénon map in the chaotic regime given by:

$$\begin{aligned} x_{n+1} &= 1 + y_n - 1.4x_n^2, \\ y_{n+1} &= 0.3x_n. \end{aligned} \tag{4}$$

We generate the 4×10^5 observations from this map and use the first 2600 points with the input $\mathbf{u} = (x, y)$ for training each reservoir oscillator with the reservoir parameter $\alpha = 0.25$. After the training stage, each reservoir oscillator has captured the underlying dynamics of the learned Hénon map. Then we address the coherence behavior of the reservoir oscillators in the Barabási-Albert (BA) network with size $N = 100$ and the coupling strength $\rho = 0.7$ [25]. According to Eqs. (1)-(3), these coupled reservoir oscillators can run autonomously whose initial values are randomly chosen. Interestingly, we find that although the initial states of the reservoir oscillators are different, they soon approach an identical profile as shown in Fig. 1(a). This is further supported by observing the standard square deviation which is defined as follows:

$$\eta = \frac{1}{N}\sum_{i=1}^{N}(\mathbf{u}_i(t) - \bar{\mathbf{u}}(t))^2, \tag{5}$$

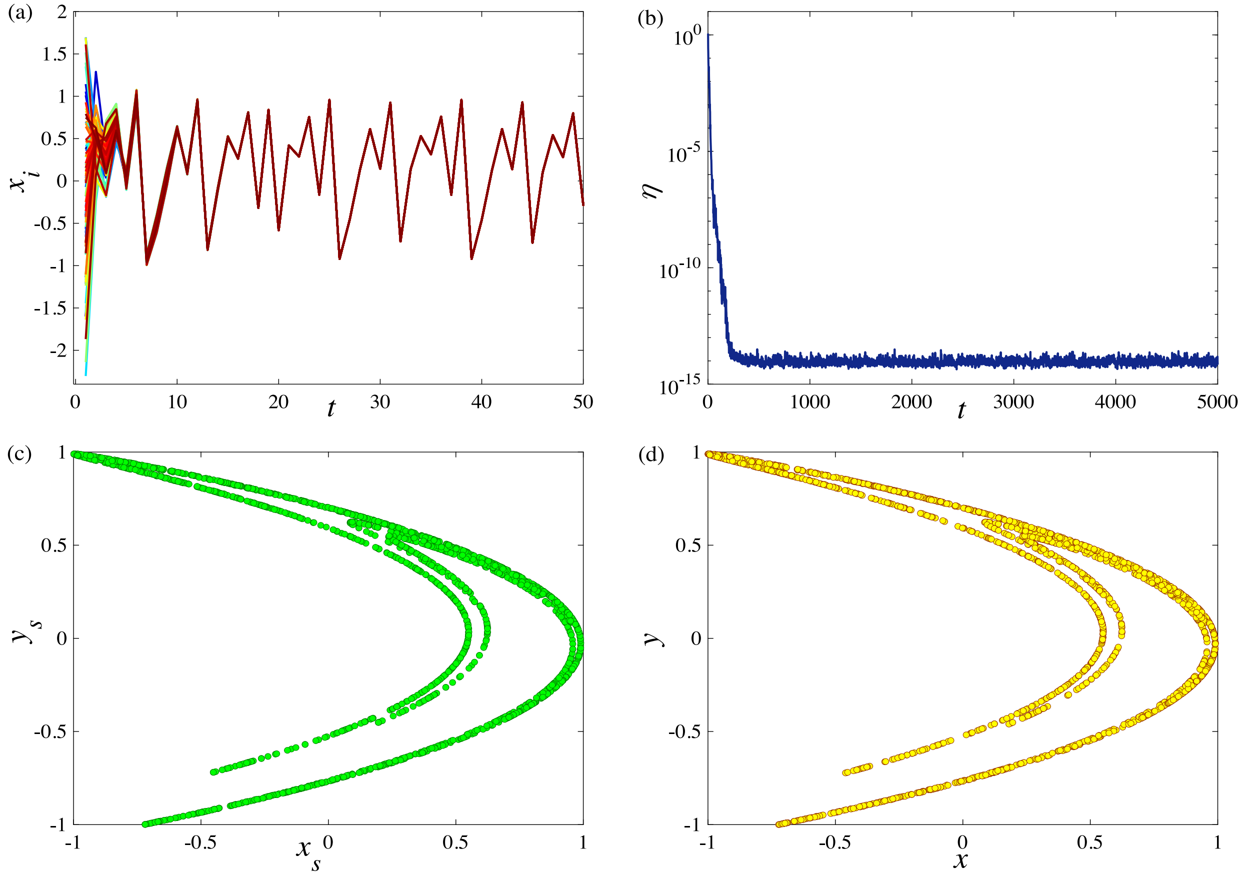

Fig. 1. (a) The x-variable of coupled reservoir oscillators as a function of time t. (b) The standard square deviation η as a function of time t. Attractors in phase space for (c) the synchronous orbit and (d) the Hénon map.

where $\bar{\mathbf{u}}(t)$ is the average value at time t. It is shown that the standard square deviation η converges to zeros, see Fig. 1(b). These findings reveal that synchronization of the reservoir oscillators can be achieved via the coupling scheme. Meanwhile, we notice that the synchronous orbit seems to present an identical profile to that of the Hénon map. Finally, we reconstruct the attractor from this synchronous state. It exhibits an almost same pattern as that from the Hénon map as illustrated in Figs. 1(c) and (d). This finding implies that the synchronous state of the reservoir oscillators has the same trajectory in phase space as that of a learned chaotic system.

3. Simulation results

3.1. Correlation dimension and recurrence time of the synchronous orbit

To address this, we now observe a classic geometrical measure known as the correlation dimension, which quantifies the self-similarity characteristics of a chaotic attractor [26]. In particular, for the m points $\{\mathbf{X}_i\}_{i=1}^m$ in phase space, its correlation integral $C(r)$ is defined as follows:

$$C(r) = \frac{1}{m^2} \sum_{i,j=1}^{m} H(r - |\mathbf{X}_i - \mathbf{X}_j|), \tag{6}$$

where $H(\cdot)$ is the Heaviside function. The correlation integral $C(r)$ quantifies how many pairs of points have a Euclidean distance less than a given threshold value r. Naturally, for a self-similarity strange attractor, a power-law relationship holds such that $C(r) \sim r^d$, where d is the correlation dimension. When utilizing the Grassberger and Procaccia algorithm for calculating correlation dimension of the synchronous orbit, we find that a clear power-law behavior emerges, see Fig. 2(a). Interestingly, it is shown that its profile on a doubly logarithmic scale almost matches that of the Hénon map. This result points out that self-similarity is preserved in the synchronous orbit.

Going beyond the geometrical perspective, we further employ recurrence time statistics to extract temporal correlation hidden in the synchronous orbit [27]. Specifically, for a given radius ϵ, we calculate the set $\Omega(\epsilon)$ of recurrence points for which $\Omega(\epsilon) = \{X_{t_i} : \| X_{t_i} - X_0 \| < \epsilon\}$. We then enumerate the recurrence times from $\Omega(\epsilon)$ such that $T = \{T(i) : T(i) = t_{i+1} - t_i\}$. Interestingly, when

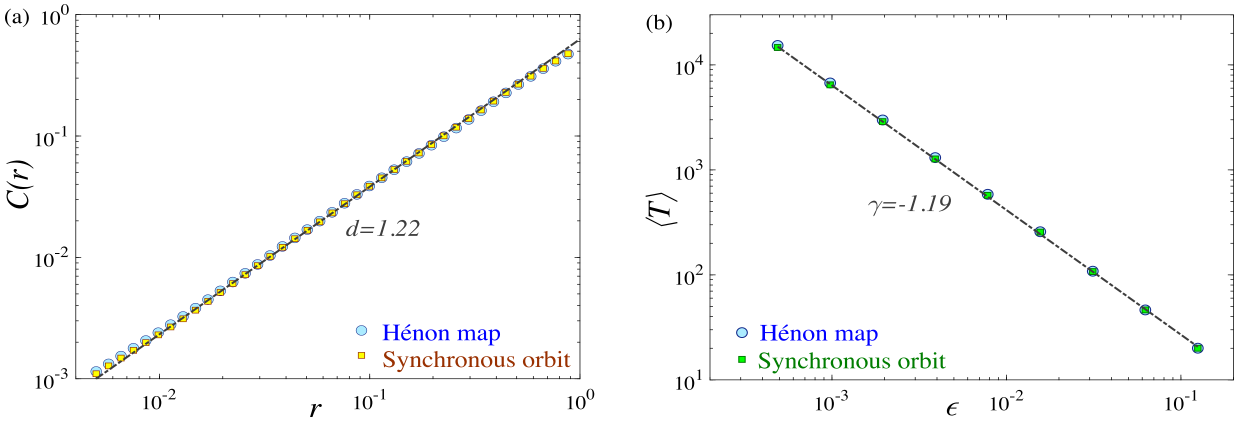

Fig. 2. Comparison of the Hénon map and the synchronous state of the reservoir oscillators with respect to (a) correlation dimension and (b) mean recurrence time.

observing the mean recurrence time $\langle T \rangle$ of the synchronous orbit, a clear scaling law emerges such that $\langle T \rangle \sim \epsilon^{\gamma}$, see Fig. 2(b). This scaling behavior is in good agreement with that of the Hénon map. These findings reveal that the synchronous orbit of the reservoir oscillators has identical geometrical and dynamical characteristics as that of a learned chaotic map of interest.

3.2. Bifurcation of the synchronous orbit

Furthermore, from Eq. (1), we notice that the "leakage" rate α is a critical reservoir parameter which controls how much history information is used in the evolution process of the reservoir oscillators. We then observe the effect of this parameter on synchronization of the previous reservoir oscillators. Interestingly, we find that different values of α admit distinct dynamics of the synchronous orbit. Specifically, when resetting $\alpha = 0.2$, the synchronous orbit displays clear periodic dynamics with period-2, see Fig. 3(a). While $\alpha = 0.231$ generates low-order periodic (period-4) orbit as shown in Fig. 3(b). After that, it will enter more complicated chaotic regimes. As shown in Fig. 3(c), the dynamical behavior of the synchronous state starts with a low-order periodic motion and then transitions to chaotic through a sequence of period-doubling bifurcations as α varies in the range $[0.19, 0.25]$. This is further supported by observing the averaged standard square deviation $\langle \eta \rangle = \sum_t \eta$, which tends to zeros as illustrated in Fig. 3(d). This intriguing bifurcation phenomenon reveals that by varying the "leakage" rate α, we can achieve distinct dynamics of the synchronous state as expected. Our finding in turn suggests that we can recover the bifurcation diagram of a dynamical system of interest via synchronization of the reservoir oscillators.

3.3. Synchronization of the reservoir oscillators upon learning the Lorenz system

We further investigate this synchronization behavior in continuous chaotic systems. In particular, we consider each reservoir oscillator learning the Lorenz system in the chaotic regime given by:

$$\begin{aligned} dx/dt &= 10(y - z), \\ dy/dt &= -xz + 60x - y, \\ dz/dt &= xy - 8/3z. \end{aligned} \tag{7}$$

By utilizing the fourth-order Runge–Kutta technique, we produce 1×10^4 data points with step size $\Delta t = 0.02$. After discarding transient, we use the first 2600 points with the input $\mathbf{u} = (x, y, z)$ for training each reservoir oscillator with the reservoir parameter $\alpha = 0.25$ and then observe the collective behaviors of the reservoir oscillators on the previous BA network with size $N = 100$ and the coupling strength $\rho = 0.6$. Similarly, a collection of coupled reservoir oscillators are clearly in identical synchrony as shown in Fig. 4(a). Hence, synchronization of the reservoir oscillators is also achieved even learning a continuous chaotic system. Moreover, when studying correlation dimension and mean recurrence time of their synchronous orbit, we find that they present identical profiles as that of the Lorenz system, see Figs. 4(b) and (c). For example, a clear power-law behavior emerges between $\langle T \rangle$ and ϵ such that $\langle T \rangle \sim \epsilon^{-1.97}$. This confirms that the synchronous orbit of the reservoir oscillators resembles the climate of the dynamical system. Furthermore, we show that this synchronization phenomenon is robust with the coupling strength ρ. When ρ is larger than 0.1, the averaged standard square deviation $\langle \eta \rangle$ equals zeros as illustrated in Fig. 4(d). Our finding reveals that synchronization of the reservoir oscillators can be achieved in a wide window of the coupling strength.

3.4. Application to speech data

Finally, we test our method on human speech data. For simplicity, we consider the Chinese vowel /i/ recorded from the adult speaker, whose analog signals are digitized by a sampling rate of 44.1 kHz and 16-bit sample resolution [28]. We select 2×10^4

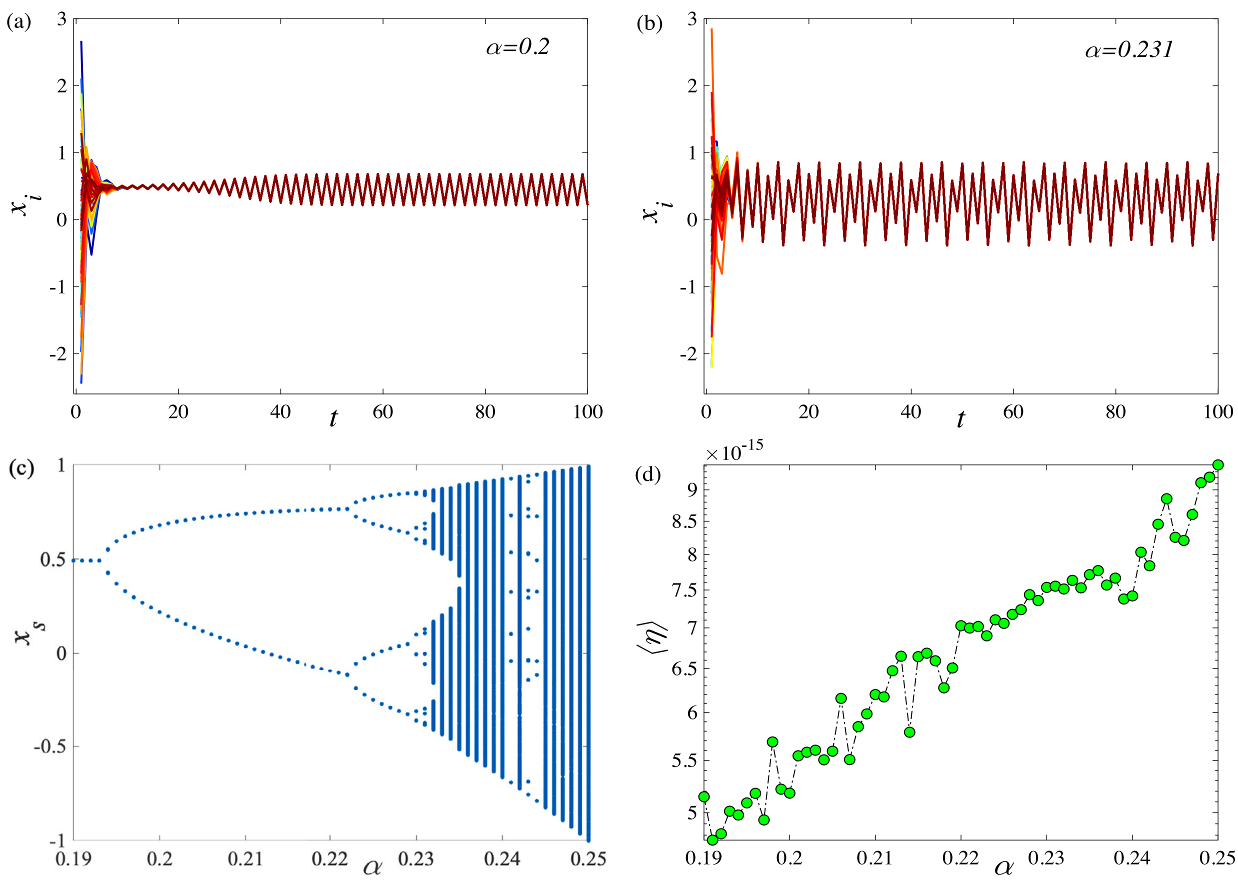

Fig. 3. The collective behaviors of the reservoir oscillators with respect to different "leakage" rates: (a) $\alpha = 0.2$ and (b) $\alpha = 0.231$. (c) The diagram of the x-variable of the synchronous orbit as a function of the "leakage" rate α. (d) The averaged standard square deviation $\langle \eta \rangle$ as a function of the "leakage" rate α.

data points (approximately 0.5 seconds) as one sample and adopt the Butterworth filter to process them. Note that the proposed method can be also shifted to more complicated speech signals. Following the same procedure we have done before, we use the first 2600 points of the speech data for training each reservoir oscillator with the reservoir parameter $\alpha = 0.2$. After the training stage, each reservoir oscillator has captured the underlying dynamics of the learned speech data, as shown in Fig. 5(a). We then observe the collective behaviors of the reservoir oscillators on the previous BA network with size $N = 100$ and the coupling strength $\rho = 0.9$. Similarly, we find that the coupled reservoir oscillators quickly approach identical synchronous orbits and the corresponding standard square deviation converges to zeros, see Figs. 5(b) and (c). Moreover, we calculate the correlation dimension of their synchronous orbit, which is nearly consistent with that of the speech data, as illustrated in Fig. 5(d). These results reveal that by virtue of the reservoir computing technique, our approach can be applied to study a great deal of fascinating synchronization phenomena for which only observational data is available.

4. Conclusions

In summary, we study synchronization in complex networks coupled with a machine learning technique. In particular, we adopt a reservoir computing approach to modeling dynamical systems and in turn this serves as a reservoir oscillator rather than a great variety of oscillators represented by the dynamical equations. By doing this, we can study a broad range of synchronization phenomena in complex systems even their analytical equations of oscillators are unknown. We design a coupled configuration of the reservoir oscillators in networks and show that synchronization can occur over a wide range of coupling strengths. Moreover, we find that its synchronous orbit has contained a number of geometrical and dynamical features such as correlation dimension and mean recurrence time, which resemble to a learned system of interest. Remarkably, by virtue of our synchronization scheme, we can uncover bifurcation phenomenon of a dynamical system of interest via its chaotic data information. Our work opens a new framework for studying synchronization phenomena in natural and man-made systems for which analytical equations are abandoned. Note that for convenience, we only study synchronization of the reservoir oscillators on a BA network. Of course, this can be applied to a great variety of other network structures, such as fractal networks [29,30], time-varying networks [8], and multilayer networks [9].

The benefit of our paradigm is three-fold. For the first time, we realize synchronization of the reservoir oscillators in complex networks. Our work reveals that the machine learning technique can be adopted to analyze synchronization phenomena. Second,

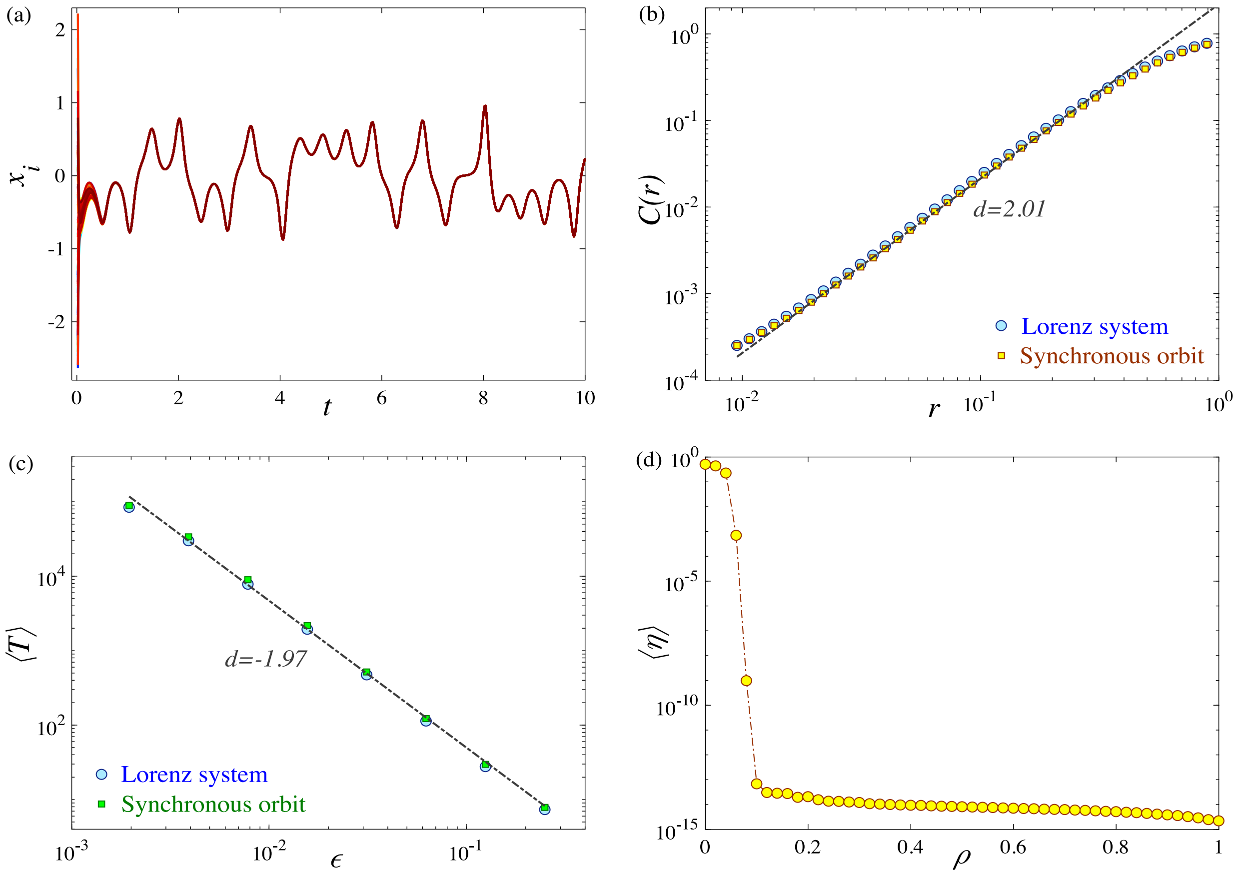

Fig. 4. (a) The x-variable of coupled reservoir oscillators as a function of time t. Comparison of the Lorenz system and the synchronous state of the reservoir oscillators in terms of (b) correlation dimension and (c) mean recurrence time. (d) The averaged standard square deviation $\langle \eta \rangle$ as a function of the coupled strength ρ.

since the reservoir oscillator has a powerful power to model dynamical systems for which only observational data is only required, our approach can be used to study and analyze a broad range of synchronization phenomena in nature and man-made systems such as biological systems, neuroscience, circadian rhythms, data mining, social sciences, and economy. Third, when adopting this data-driven technique for achieving synchronization, we can obtain a number of intriguing findings. For example, we can recover the bifurcation phenomenon of a dynamical system via its chaotic data information.

CRediT authorship contribution statement

Tongfeng Weng: Conceptualization, Methodology, Software. **Xiaolu Chen:** Data curation, Writing – original draft. **Zhuoming Ren:** Investigation, Visualization. **Huijie Yang:** Supervision. **Jie Zhang:** Software, Validation. **Michael Small:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (NNSFC) under Grant Nos. 11805128 (T. Weng), 61803137 (Z. Ren), and 61973086 (J. Zhang); Shanghai Municipal Science and Technology Major Project (No. 2018SHZDZX01) and

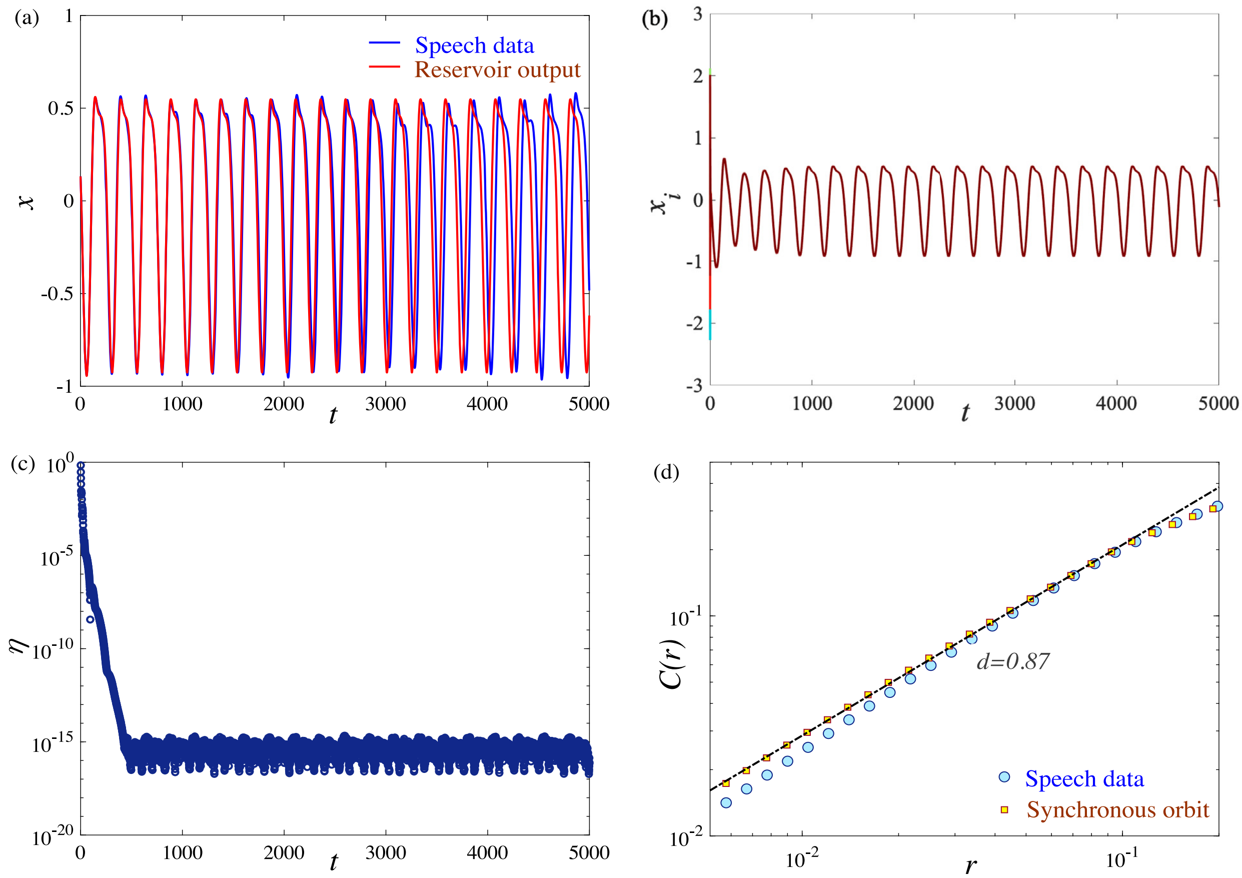

Fig. 5. (a) Prediction output of the trained reservoir computer overlaid with actual speech data. (b) The x-variable of coupled reservoir oscillators as a function of time t. (c) The standard square deviation η as a function of time t. (d) Comparison of speech data and the synchronous state of the reservoir oscillators in terms of correlation dimension.

ZJ lab; the Shanghai project for construction of top disciplines under Grant No. USST-SYS-01 (H. Yang). T. Weng acknowledges a Xihu Scholar award from Hangzhou City and the Hangzhou Normal University Starting Fund (4135C50220204098).

References

[1] A.T. Winfree, Biological rhythms and the behavior of populations of coupled oscillators, J. Theor. Biol. 16 (1967) 15–42.
[2] L. Glass, Synchronization and rhythmic processes in physiology, Nature (London) 410 (2001) 277–284.
[3] M. Rohden, A. Sorge, M. Timme, D. Witthaut, Self-organized synchronization in decentralized power grids, Phys. Rev. Lett. 109 (2012) 064101.
[4] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, C.S. Zhou, Synchronization in complex networks, Phys. Rep. 469 (2008) 93–153.
[5] F.A. Rodrigues, T.K. Peron, P. Ji, J. Kurths, The kuramoto model in complex networks, Phys. Rep. 610 (2016) 1–98.
[6] M. Barahona, L.M. Pecora, Synchronization in small-world systems, Phys. Rev. Lett. 89 (2002) 054101.
[7] X.F. Wang, Complex networks: topology, dynamics and synchronization, Int. J. Bifurc. Chaos 12 (2002) 885–916.
[8] V. Kohar, P. Ji, A. Choudhary, S. Sinha, J. Kurths, Synchronization in time-varying networks, Phys. Rev. E 90 (2014) 022812.
[9] X.Y. Zhang, S. Boccaletti, S.G. Guan, Z.H. Liu, Explosive synchronization in adaptive and multilayer networks, Phys. Rev. Lett. 114 (2015) 038701.
[10] M.R. Tinsley, S. Nkomo, K. Showalter, Chimera and phase-cluster states in populations of coupled chemical oscillators, Nat. Phys. 8 (2012) 662–665.
[11] L.M. Pecora, F. Sorrentino, A.M. Hagerstrom, T.E. Murphy, R. Roy, Cluster synchronization and isolated desynchronization in complex networks with symmetries, Nat. Commun. 5 (2014) 4079.
[12] B.M. Adhikari, C.M. Epstein, M. Dhamala, Localizing epileptic seizure onesets with granger causality, Phys. Rev. E 88 (2013) 030701(R).
[13] S.H. Strogatz, D.M. Abrams, A. McRobie, B. Eckhardt, E. Ott, Crowd synchrony on the millennium bridge, Nature (London) 438 (2005) 43–44.
[14] A. Roxin, H. Riecke, S.A. Solla, Self-sustained activity in a small-world network of excitable neurons, Phys. Rev. Lett. 92 (2004) 198101.
[15] A.A. Mohammed, R. Minhas, Q.M. Wu, M.A. Sid-Ahmed, Human face recognition based on multidimensional pca and extreme learning machine, Pattern Recognit. 44 (2011) 2588–2597.
[16] D. Silver, J. Schrittwieser, K. Simonyan, I. Antonoglou, A. Huang, Mastering the game of go without human knowledge, Nature (London) 550 (2017) 354–359.
[17] H. Jaeger, H. Haas, Harnessing nonlinearity: predicting chaotic systems and saving energy in wireless communication, Science 304 (2004) 78–80.
[18] J. Pathak, B. Hunt, M. Girvan, Z.X. Lu, E. Ott, Model-free prediction of large spatiotemporally chaotic systems from data: a reservoir computing approach, Phys. Rev. Lett. 120 (2018) 024102.
[19] T.F. Weng, H.J. Yang, C.G. Gu, J. Zhang, M. Small, Synchronization of chaotic systems and their machine-learning models, Phys. Rev. E 99 (2019) 042203.
[20] J. Pathak, Z.X. Lu, B.R. Hunt, M. Girvan, E. Ott, Using machine learning to replicate chaotic attractors and calculate lyapunov exponents from data, Chaos 27 (2017) 121102.

[21] X.L. Chen, T.F. Weng, H.J. Yang, C.G. Gu, J. Zhang, M. Small, Mapping topological characteristics of dynamical systems into neural networks: a reservoir computing approach, Phys. Rev. E 102 (2020) 033314.
[22] J.J. Jiang, Y.C. Lai, Model-free prediction of spatiotemporal dynamical systems with recurrent neural networks: role of network spectral radius, Phys. Rev. Res. 1 (2019) 033056.
[23] P.R. Vlachas, J. Pathak, B.R. Hunt, T.P. Sapsis, M. Girvan, E. Ott, P. Koumoutsakos, Backpropagation algorithms and reservoir computing in recurrent neural networks for the forecasting of complex spatiotemporal dynamics, Neural Netw. 126 (2020) 191–217.
[24] M. Lukoševicius, H. Jaeger, Reservoir computing approaches to recurrent neural network training, Comput. Sci. Rev. 3 (2009) 127–149.
[25] A.-L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (1999) 509–512.
[26] P. Grassberger, I. Procaccia, Characterization of strange attractors, Phys. Rev. Lett. 50 (1983) 346–349.
[27] J.B. Gao, Recurrence time statistics for chaotic systems and their applications, Phys. Rev. Lett. 83 (1999) 3178.
[28] J. Sun, J. Zhang, M. Small, Extension of the local subspace method to enhancement of speech with colored noise, Signal Process. 88 (2008) 1881–1888.
[29] B. Wang, J. Zhu, D. Wei, The self-similarity of complex networks: from the view of degree–degree distance, Mod. Phys. Lett. B 35 (2021) 2150331.
[30] J. Zhu, D. Wei, Analysis of stock market based on visibility graph and structure entropy, Physica A 576 (2021) 126036.