Chaos, Solitons and Fractals 166 (2023) 112880

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

journal homepage: www.elsevier.com/locate/chaos

Regulating clustering and assortativity affects node centrality in complex networks

Xing-Zhang Wen, Yue Zheng, Wen-Li Du, Zhuo-Ming Ren *

Research Center for Complexity Sciences, Alibaba Business School, Hangzhou Normal University, Hangzhou, China

ARTICLE INFO

Keywords:
Node centrality
Clustering coefficient
Assortative coefficient
Null model
Complex networks

ABSTRACT

The limitations of classical node centralities such as degree, closeness, betweenness and eigenvector are rooted in the network topology structure. For a deeper understanding, we regulate the basic network topology structure clustering and assortative coefficient to study the effect on these four classical node centralities. To observe the structural diversity of the complex network, we firstly construct two types of the growing scale-free networks with tunable clustering coefficient and assortative coefficient respectively, and simulate three types of null models on ten real networks to adjust cluster and assortativity. The results indicate that the impact of varying cluster and assortativity on node centrality in complex networks is obvious. We should pay more attention to the network topology when selecting node centralities as identifying the significance or influence of nodes in complex networks.

1. Introduction

Node centrality has some serious flaws [1,2]. In particular, we show that, depending on the network structure. The classical centrality includes the degree centrality as the number of neighbors a node connects with, the closeness centrality [3] as the reciprocal of the sum of the geodesic distances to all other nodes, betweenness centrality [4] as the number of shortest paths through a certain node, and the eigenvector centrality [5] as the component of the eigenvector to the largest eigenvalue of the adjacency matrix. Lately, a lot of works try to design efficient algorithms that outperform the classical centrality methods. Some algorithms focus on directly modifying or extending the basic centrality measures including degree [6–8], closeness [9], and betweenness [10,11]. Some others try to cut down the computational complexity of eigenvector [12,13]. Node centralities are well-known methods for quantifying the influence of nodes, as well as ranking nodes in complex networks [14–16]. For instance, we can hinder spreading in the case of diseases or accelerate spreading in the case of information dissemination. The above classical or extended centralities could manifest different spreader topology in a network, which leads to different efficacy and applicability for ranking the influence of the spreaders. The centrality method will generate a ranking list for nodes. In principle, the ranking from an effective ranking method should be as close as possible to the ranking based on the real spreading process. Node centrality measurements are based on characterizing the network topology structure in a certain perspective. Changing the network topology structure would affect the accuracy of the node centrality.

To be known as a hot research field of complex networks, the identification of node influence is of great theoretical and practical importance, but we usually neglect the impact of varying network structure. Network structure often affects the functions of nodes such as spreading, reputation, synchronization, and controllability [17]. There are many studies focusing on the impact of some network structure on the function of nodes. For example, the analysis of influence and susceptibility together with network structure reveals that influential individuals are less susceptible to influence than non-influential individuals and that they cluster in the network while susceptible individuals do not [18]. The contribution of a node to the spreading behavior is not uniquely determined by the structure of the system but it is a result of the interplay between dynamics and network structure [19]. Statistical analysis also shows that social networks typically show a high clustering, or local transitivity [20]: If person A knows B and C, then B and C are likely to know each other. The behavior spread farther and faster across clustered-lattice networks than across corresponding random networks [21]. Social messages have more impact than informational messages and 'weak ties' are much less likely than 'strong ties' to spread behavior via the social network [22]. In addition, the assortativity which is also a standard tool for analyzing network structure and has a simple interpretation [23,24]. Assortativity r ranges from -1 (i.e. disassortativity) to 1 (i.e. assortativity). If $r > 0$, high degree nodes tend to connect to other high degree nodes. Otherwise, high degree nodes tend to connect to low degree nodes. In practical

* Corresponding author.
E-mail address: zhuoming.ren@hznu.edu.cn (Z.-M. Ren).

https://doi.org/10.1016/j.chaos.2022.112880
Received 15 January 2022; Received in revised form 28 October 2022; Accepted 3 November 2022
Available online 22 November 2022

application scenarios, mostly complex networks extracted from society, biology, information, technology appear a very diverse range of structure topology. We can give an evidence of ten real networks in Table 1 of the context. Table 1 shows that the number of nodes ranges from dozens to tens of thousands, the edges range from tens to hundreds of thousands, assortativity from negative to positive, cluster are from 0 to 0.6. Some of network models can also generate the growing scale-free networks such as with tunable clustering or with tunable assortative coefficient. Furthermore, the null models by randomizing real networks could fix some of the structural properties including clustering and assortative coefficient to the values observed in original networks, many other properties appear as statistical consequences of these fixed observables [25,26].

In order to fill in the blanks, we will focus on the impact of varying cluster and assortativity on the four classical node centralities. For a deeper understanding, the Gini index, the coefficient of variation and correlation are used to investigate the four classical node centralities when we regulate the basic network topology structure clustering and assortative coefficient. We firstly construct a lot of growing scale-free networks with tunable clustering and assortative coefficient, then ten real networks are independently randomized by three types of null models to observed impact of varying topology on node centrality in complex networks. The randomization methodology can assess the significance of the structural property in complex networks [27]. For example, Ren et al. proposed a time-respecting null model that preserved both the network's degree sequence and the time evolution of individual nodes' degree values [28]. This null model is able to factor out the effect of the system's temporal patterns on its structure. Furthermore, the null models of real networks play a critical role in many research fields such as the detection of communities [29], network motifs [30], network superfamilies [31], assortativity [32] or revising assortativity [33].

2. Methods and materials

2.1. Node centrality

We first describe four well-known centralities considered in this paper. Normally, an undirected network $G = (V, E)$ with $n = |V|$ nodes and $e = |E|$ links could be represented by an adjacent matrix $\Omega = \{a_{ij}\} \in R^{n,n}$, where $a_{ij} = 1 (i \neq j)$ if node i and node j are connected, and $a_{ij} = 0$ otherwise. Degree centrality is the simplest one, which is defined as the number of connections of a node.

$$kc(i) = \sum_{j=1} a_{ij}. \tag{1}$$

It is reasonable to assume the nodes with many connections have stronger influence than those with few connections. In most cases, the degree is a powerful index for ranking nodes' influence for its low computational cost. Closeness centrality [3] of node i is defined as the reciprocal of the sum of the geodesic distances to all other nodes,

$$cc(i) = \frac{N-1}{\sum_{i \neq j} d_{ij}}, \tag{2}$$

where N is the number of nodes, and d_{ij} is the geodesic distances from i to j. Nodes with high closeness have short distances to others, and thus generally more influential in spreading. In the case of information diffusion, people usually consider individuals with high closeness value as being well-positioned to obtain novel information early. Likewise, nodes with high closeness value in a epidemic network are positioned to infect others easily. Betweenness centrality [4] is defined as follows: consider any node pair (s, t) and σ_{st} is the total number of shortest paths between these two nodes. If the number of the shortest paths passing through node i is denoted by σ^i_{st}, then the betweenness centrality of node i is given by,

$$bc(i) = \sum_{s \neq t} \frac{\sigma^i_{st}}{\sigma_{st}}. \tag{3}$$

Nodes with high betweenness centrality may have considerable influence in a network since a lot of shortest paths are passing through them. The closeness centrality and the betweenness centrality could effectively quantify the influence of node, but they are with high computational complexity due to calculating the shortest paths between all pairs of nodes in a network. Eigenvector centrality is that a node's importance is not only determined by itself, but also affected by its neighbors' importance [5]. A node connecting to important nodes will make itself also important. With this idea, the eigenvector centrality of node i can be defined as:

$$ec(i) = \frac{1}{\lambda} \sum_{j=1} a_{ij} e(j). \tag{4}$$

where λ is not more than largest eigenvalue. Actually, in the matrix theory,

$$Ae = \lambda e. \tag{5}$$

Clearly, due to this recursive property, eigenvector centrality can reflect the global features of the network.

2.2. Centrality evaluation

For a network, a sequence will be generated through the node centrality algorithm. But when the network structure changes, the sequence will also change. We will use Gini coefficient, the coefficient of variation and correlation to analyze the effect of varying cluster and assortativity on node centrality in complex networks. *Gini coefficient* measures the inequality among values of a frequency distribution, and calculates the area between the Lorenz curve and a hypothetical line of absolute equality, which is expressed as a percentage of the maximum area under the line. Thus, a gini coefficient of zero expresses perfect equality, where all values are the same. A gini coefficient of 1 expresses maximal inequality among values. *Coefficient of variation* (CV) is a statistical measure of the dispersion of data points in a data series around the mean, which relates the mean and standard deviation by expressing the standard deviation as a percentage of mean. The benefit of standard deviation is an absolute measure which explains the dispersion in the same unit as original data. It is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different from one another.

3. Results

3.1. Effects of variable network clustering in scale free networks

Node centralities are based on characterizing the network topology structure in a certain perspective. Changing the network topology structure would affect the performance of the node centrality. In this section, we employ the Holme–Kim(namely HK) model [34] to construct scale free networks with tunable clustering to analyze the effects of variable network clustering to the four classical centralities including degree centrality, closeness centrality, betweenness centrality and eigenvector centrality. The HK model is introduced as follows, (1). Initial condition: the network consists m_0 initial fully connected nodes; (2). Preferential attachment (PA): at each time step t, a new single node j is added to the network. In the meanwhile, the new node j selects m other existed nodes to connect. The probability that a new link will connect j to an existing node i is proportional to the number of links that i already, $P(j \to i) = \frac{k_i}{\sum_l k_l}$. (3). Triad formation (TF): If an link between j and i was added in the previous PA step, then add one more edge from j to a randomly chosen neighbor of i with the probability $p = M_t/(m-1)$. Where $0 \leq p \leq 1$, thus the parameter $0 \leq M_t \leq (m-1)$. It should be noted that HK model reduces to the original scale-free network named BA model [35] when $M_t = 0$.

The basic parameter values are that $N = 10,000$, $m = m_0 = 3$, and the clustering coefficient with different parameters M_t is from 0

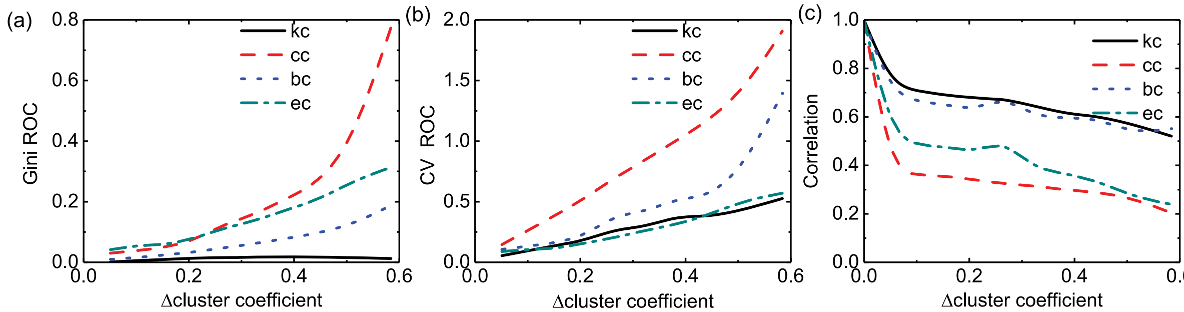

Fig. 1. Four centrality measures in scale free networks with tunable clustering. It is noted that the assortative coefficient of the networks generated by all tunable parameters is that the mean value equals −0.049, and the standard deviation equals 0.008. (a) ROC of the gini index. (b) ROC of the coefficient of variation. (c) Correlation.

to 2. For the basic attributes of the network, the degree distribution of the network conforms to the scale-free characteristic. The assortative coefficient of the network is consistent with that of BA model. That is, the assortative coefficient of the network generated by all parameters is that mean value equals −0.049, standard deviation equals 0.008. And we know that the BA network has very weak cluster, and our experimental measurement is that mean value is −0.038, standard deviation equals 0.004.

Through triad formation, the cluster of the HK model continues to increase. We use the BA network as the baseline to analyze the variation of the four centrality in the HK network as shown in Fig. 1. Fig. 1a shows the gini ROC in the tunable cluster networks. We can see that as the network cluster coefficient increases, the gini ROC of the degree centrality is closed to zero, which is to say the degree centrality consistent with that of BA, while the gini ROC of closeness centrality continues to increase, and the value is the largest, followed by betweenness centrality and the eigenvector centrality. It shows that the cluster is increased, the inequality of the closeness centrality is higher, and the degree centrality of inequality is basically unchanged. Fig. 1b shows change of CV ROC when we adjust the cluster. We can see that as the network cluster coefficient increases, and CV ROC has also been rising. Comparing the four centrality indicators, CV ROC of closeness centrality becomes the highest, followed by that of betweenness, while CV ROC of degree and eigenvector rises slowly. In addition, although the gini coefficient of the degree centrality does not change with the cluster coefficient, the coefficient of variation has changed significantly. Fig. 1c shows the correlation of the node centrality series before and after changing the clustering. We can see that as the network cluster coefficient increases, the correlation of the corresponding centrality becomes low.

3.2. Effects of variable network assortativity in scale free networks

Next, we investigate the performance of the centrality for a growing scale-free network model with tunable assortative coefficient. The growing scale-free network model with tunable assortative coefficient (namely TASF model) [36] is defined as: (1). The newly added node connects to the existing node i preferentially. This step is described as the same as Initial condition and Preferential attachment; (2). This added node then selects a neighbor node s of the node i with probability $k^{\alpha}(s)/\sum_{j\in\Gamma_i} k^{\alpha}(j)$, where α is the tunable parameter and Γ_i is the neighbor node set of node i.

We set basic parameter values as $N = 5000$ in each network, $m = m_0 = 3$, $M_t = 2$, and α is the tunable parameter. For the basic properties of the network, the degree distribution of the network conforms to the scale-free characteristic. The cluster correlation of the network is consistent with that of HK, that is, the cluster of the network generated by all parameters is that mean value equals 0.447, standard deviation equals 0.112. And we know that the assortative coefficient of the HK network is basically the same as that of BA, and our experimental measurement is that mean value equals −0.047, standard deviation equals 0.004. After through the step of adjusting assortative coefficient, we use the HK network as the baseline to analyze the gini ROC and CV ROC of the four centralities in the TASF network.

Fig. 2a shows the gini ROC with the tunable assortativity from the negative to the positive. We can see that as the negative correlation network to the positive correlation network, the gini ROC of degree centrality and betweenness centrality drops all the way, while that of closeness centrality and eigenvector centrality continues to increase. This shows that in the more negative correlation networks, the gini index of degree centrality and betweenness centrality is larger than the baseline, and as the negative correlation turns to the positive correlation, the two are getting closer to the baseline, and then smaller than the baseline. While closeness centrality and eigenvector centrality are opposite in the more negatively correlated network, the gini index of the two is smaller than the baseline, and as the negative correlation turns to the positive correlation, the two are getting closer to the baseline, and then larger than the baseline. Fig. 2b shows CV ROC with the tunable assortativity from the negative correlation to the positive correlation. We can see that as the negative correlation networks turn to the positive correlation, CV ROC of degree centrality and betweenness centrality has dropped, while CV ROC of closeness centrality and eigenvector centrality is closed to 0, that is, the two is basically consistent with the baseline. This shows that in the more negative correlation network, the gini index of degree centrality and betweenness centrality is larger than the baseline, and as the negative correlation turns to the positive correlation, the two are then smaller than the baseline, while closeness centrality and eigenvector centrality are opposite, the two is close to the baseline. Fig. 2c shows the correlation of the node centrality series before and after changing the assortativity. We can see that as the change of network assortativity coefficient increases, the correlation of the centrality also becomes low. In summary, when we adjust assortativity from the negative to the positive, we can find that degree centrality and betweenness centrality have the same the trend of change as well as closeness centrality and eigenvector centrality.

3.3. Effects of null models on ten real networks to adjust clustering and assortativity

Fixing some of the structural properties of network models to their values observed in real networks, many other properties appear as statistical consequences of these fixed observables. Here we employ the null models, a complete set of basic characteristics of the network structure, to study the statistical dependencies between different network properties. The null model is simple and easy to operate. It does not need to understand and apply complex mathematical formulas, will not produce self-loops and re-edge phenomena, and can accurately maintain some physical properties of the real networks. By freely adjusting the microscopic characteristic parameters such as the average degree, matching clustering coefficient and assortative coefficient, the structural diversity of the test network can be effectively observed. There are usually different order models according to different constraints.

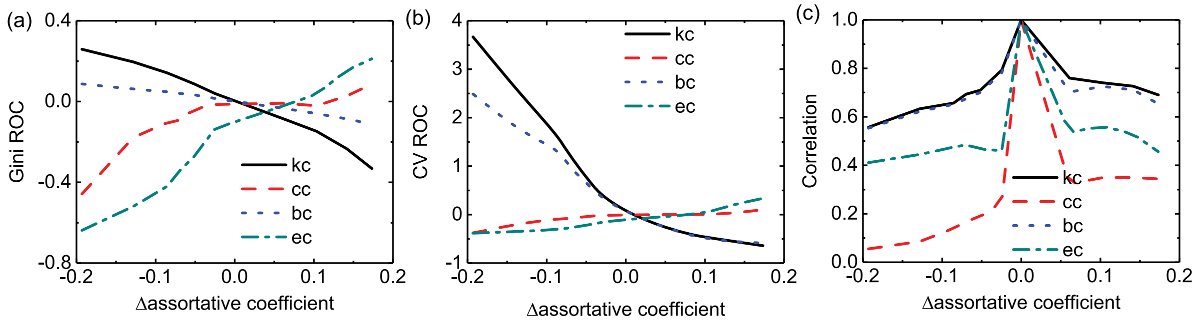

Fig. 2. Four centrality measures in the scale-free network model with tunable assortative coefficient. The cluster coefficient of these tunable assortative networks is that the mean value equals 0.447, and the standard deviation equals 0.112. (a) ROC of the gini index. (b) ROC of the coefficient of variation. (c) Correlation.

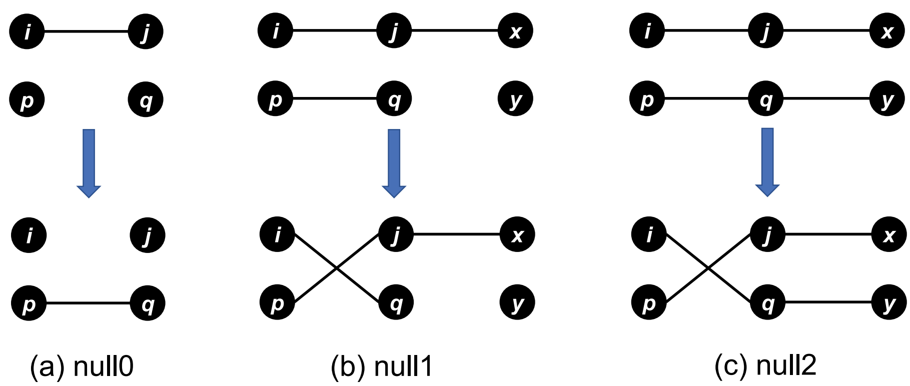

Fig. 3. Three types of the null model. (a) The zero-order null model (namely null0) is that, each time, randomly select an edge $e(i,j)$ in the original network, and then randomly select two nodes p, q. If there is no connected edge between p and q, then delete the original network edge $e(i,j)$ and create $e(p,q)$. (b) The first-order null model (namely null1) is as follows, each time the two edges in the original network are randomly selected as $e(i,j)$ and $e(p,q)$, there are only two edges in the four nodes i, j, p, q, then delete two edges $e(i,j)$ and $e(p,q)$, and create two new edges $e(i,q)$ and $e(p,j)$. (c) The second-order null model (namely null2) is that just add another constraint on the first-order null model, that is to say, nodes i and p (or j, q) have the same degree.

We apply three common null models, and the construction process is shown in Fig. 3. The zero-order null model(namely null0) has the same number of nodes and the same average degree as the original network. The specific process is as follows, each time, randomly select an edge $e(i,j)$ in the original network, and then randomly select two nodes p, q. If there is no connected edge between p and q, then delete the original network edge $e(i,j)$ and create $e(p,q)$. The first-order null model(namely null1) not only has the same number of nodes and the same average degree as the original network, but more importantly has the same node degree distribution $p(k)$. The degree distribution refers to the distribution of the probability or number of node degrees in the original network. The specific process is as follows, each time the two edges in the original network are randomly selected as $e(i,j)$ and $e(p,q)$, there are only two edges in the four nodes i, j, p, q, then delete two edges $e(i,j)$ and $e(p,q)$, and create two new edges $e(i,q)$ and $e(p,j)$. The second-order null model(null2) has the same number of nodes and the same joint degree distribution as the original network. The joint degree distribution refers to the number of degrees (probabilities) of the nodes connected to each end of each edge. The specific process is just to add another constraint on the first-order null model, that is to say, nodes i and p (or j, q) have the same degree.

We understand that null models of real networks can change the structure of the network, but how will the four centralities change in different three null models? We then use 10 real networks according to the simulation of three null models, and the network basic properties are shown in Table 1. These ten networks are diverse including social networks such as Karate [37], Vote [38], PGP [39], Email [40], biological networks like Metabolic [41], information network like USAir [42], As [43], P2P [43], S838 [31], collaboration networks like Jazz [44]. The number of nodes ranges from dozens to tens of thousands, and the edges ranges from dozens to hundreds of thousands. Assortative coefficient is from negative to positive. Clustering coefficient is from 0 to 0.6.

Table 1
Topological features of ten real networks considered. N is the number of nodes. E is the number of edges. r is assortative coefficient. c is clustering coefficient.

No.	Name	N	E	r	c
1	Karate	34	78	−0.476	0.571
2	Jazz	198	5,484	0.020	0.617
3	Metabolic	453	4,596	−0.212	0.646
4	S838	512	819	−0.030	0.05465
5	Email	1,133	5,451	0.078	0.220
6	USAir	1,332	2,126	−0.209	0.625
7	As	6,201	12,170	−0.181	0.253
8	P2P	6,301	20,778	0.036	0.011
9	Vote	7,066	100,736	−0.083	0.142
10	PGP	10,680	24,316	0.238	0.266

After 50 independent experiments of the null models of each real network, let us look at the changes of cluster and assortative in ten real networks as shown in Fig. 4. Fig. 4a shows the ROC of cluster. The null model prompts to reduce the cluster of the network. Among three kinds of null models, null0 has the strongest ability to reduce cluster, and null1 and null2 are weakened. Fig. 4b shows the ROC of assortative. We can see that under three null models, the change of assortative is different. In null0, the change is small in PGP, and the change is large in Vote; In null1, the change is small in Metabolic, but changes greatly like jazz and vote; In null2, it is almost unchanged in USAir, Metabolic, As, Vote, but changes greatly in S838. In general, in null models, the network assortative varies.

Next, we will analyze the effects of null models of real networks on the four centralities of degree, closeness, betweenness, and eigenvector.

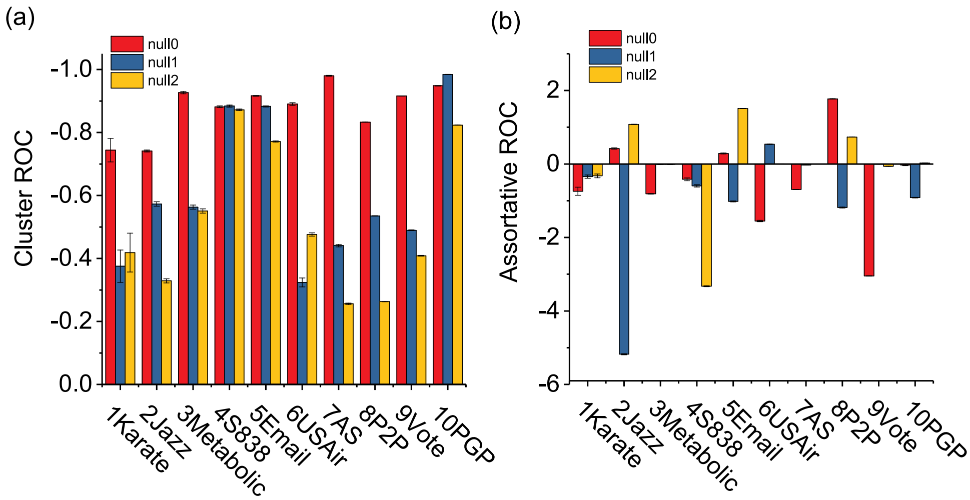

Fig. 4. The clustering coefficient and assortative coefficient in ten networks after simulating three orders of null models. (a) Clustering coefficient ROC. (b) Assortative coefficient ROC.

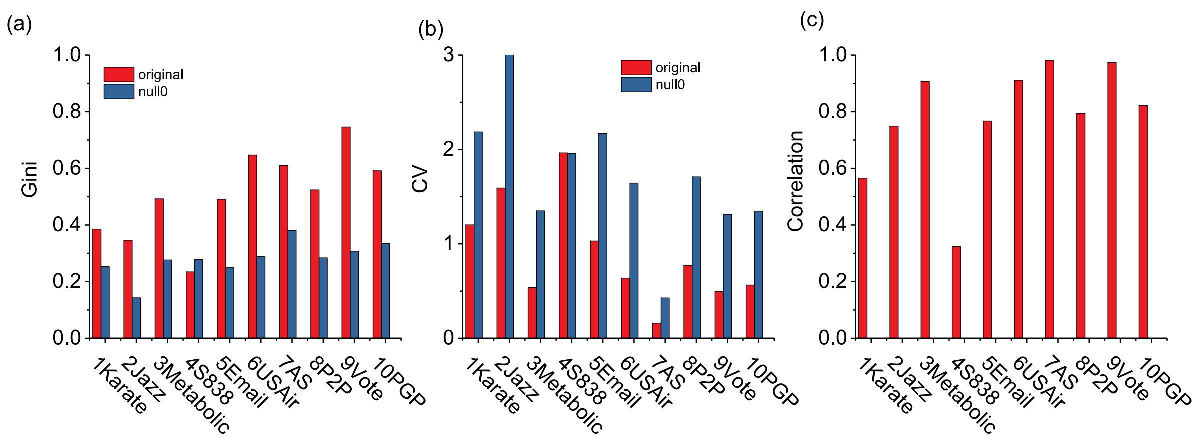

Fig. 5. Variation of degree centrality in 10 networks after simulating null0 model. We compare the degree centrality between original networks and randomizing networks by null0 model. (a) ROC of the gini index. (b) ROC of the coefficient of variation. (c) Correlation. It is noted that each node degree may change in null0, while degree of each node in the null1 and null2 will not change, so the network's degree gini index and coefficient of variation in null1 and null2 are unchanged.

Firstly, look at the results of degree as shown in Fig. 5. Each node degree may change in null0, while degree of each node in the null1 and null2 will not change, so the network's degree gini index and coefficient of variation in null1 and null2 are unchanged. Fig. 5a shows that the range of the gini index of ten original networks is between 0.2 and 0.8. After null0 simulation, except for one jazz network whose gini index is less than 0.2, the others are aggregated and then between 0.2 and 0.4, indicating that passing null0 can make the gini index within a relatively small range. Fig. 5b is given the coefficients of variation of the original network and null0. We can see that the coefficient of variation in the original network is less than 2, but after the null0, except for the jazz network is close to 4, the other changes are not large. Fig. 5c is given the correlation between the original network and null0. We can see that the correlation of the networks like karate and s838 is lower than 0.6. While correlation of networks like metabolic, USAir, as, vote is close to 1, which means the ranking of the node degree centrality series between the original network and null0 are mostly same.

We continue to analyze the variation of the closeness in the original network and the three null models as shown in Fig. 6. Let us look at the change of the gini index as shown in Fig. 6a. In the original networks, the gini index of closeness is around 0.08, and it is smaller than 0.08 in null0, while in null1 and null2, there is little change from the original. Therefore, in general, the value of closeness in the original network and the three null models are relatively small, indicating that the inequality of closeness itself is relatively small. In Fig. 6b, we see that coefficients of variation of both the original network and the three null models is greater than 5. Especially the null0 coefficient of variation in jazz exceeds 30. Compared with the coefficient of variation in the original networks, null0 becomes larger, and null1 and null2 remain unchanged. Fig. 6c shows that the correlation of the closeness centrality series between the original network and three null models. We can see the correlation of the closeness centrality series in all networks is low after simulating first order null models. In addition, if we simulate higher order null model to the network, we can get more similar network as the original network, so the correlation of the closeness centrality series becomes large.

Fig. 7 shows the variation coefficient of betweenness between the original network and three null models. Fig. 7a also shows the situation of the original network and three null models. It is worth noting here that there are two networks, the betweenness of the gini index in As network is the largest in ten networks, but it becomes smaller in null0. Another network is that Jazz's gini index is only close to about 0.3 in null0. In general, the gini index is getting smaller in null0, and there is basically no change in null1 and null2. The variation of betweenness is shown in Fig. 7b. We can see that there are some difference from the gini index. That is, Jazz has the largest coefficient of variation in null0, while the as network has a coefficient of variation near zero. In other networks, the three null models and origin have little change. In

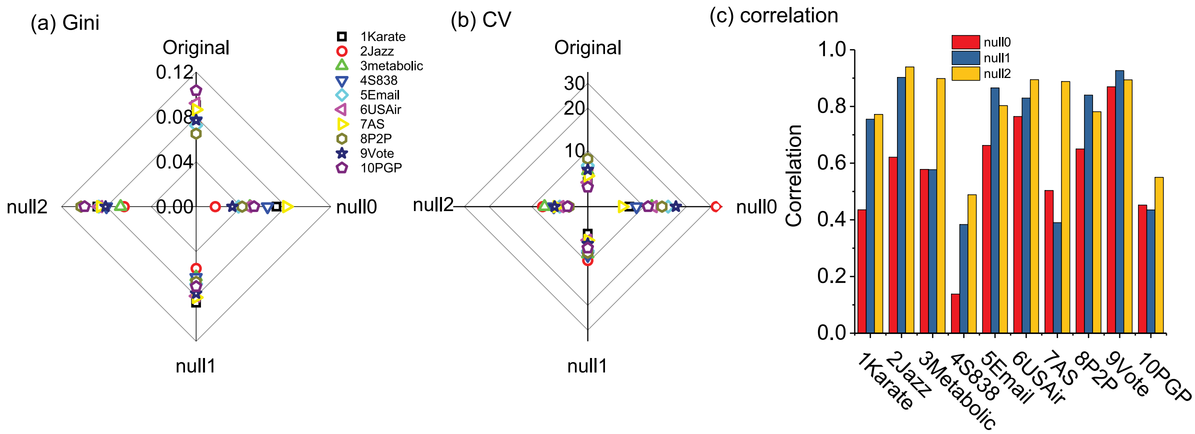

Fig. 6. Variation of closeness centrality in 10 networks after simulating three null models. We compare the closeness centrality between original networks and the randomization of networks by three orders of null models. (a) The gini value of closeness in the original network and the three null models are relatively small, indicating that the inequality of closeness itself is relatively small. (b) Compared with the coefficient of variation in the original networks, null0 becomes larger, but null1 and null2 seem unchanged. (c) Correlation of closeness centrality between the original networks and three null models.

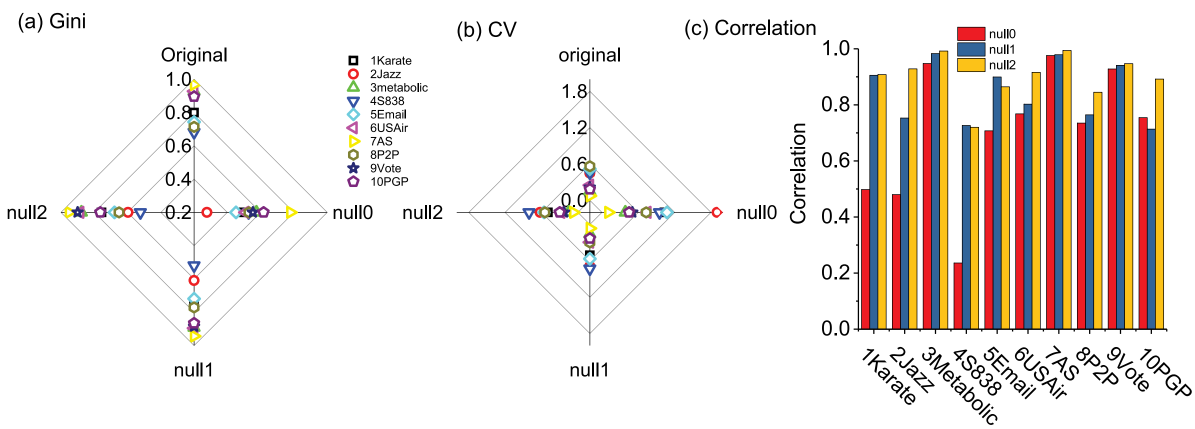

Fig. 7. Variation of betweenness centrality in 10 networks after simulating three null models. We compare the betweenness centrality between original networks and the randomization of networks by three orders of null models. (a) The gini index is getting smaller in null0, but there is basically no change in null1 and null2. (b) The coefficient of variation of betweenness in the original network and the three null models are relatively small. (c) Correlation of betweenness centrality.

Fig. 7c, we see that the correlation of the betweenness centrality series between the original network and three null models which displays the same results as Fig. 6c.

The last one we will analyze is the eigenvector centrality as shown in Fig. 8. As one can be seen in Fig. 8a, the gini coefficients of the eigenvector centrality in 10 original networks range from 0.2 to 1. After the simulation of three null models, the change of the gini value is not large, and the gini value range is still wide. The coefficient of variation is shown in Fig. 8b. We can see that Jazz has the largest coefficient of variation in null0, while the AS network has a coefficient of variation near zero. In other networks, the three null models have little change. The correlation of eigenvector is shown in Fig. 8c, we see that the correlation of the eigenvector centrality series between the original network and three null models which describes the same results as closeness and betweenness.

In summary, we can find that the gini coefficient of closeness centrality is the smallest, betweenness centrality is the largest, and the other two are widely distributed. After the simulation of null models, the gini index of the four centralities will become smaller, especially after null0, this trend is most obvious. The coefficient of variation of the four centralities is just the opposite. We can find that the coefficient of variation of closeness is the largest, betweenness is the smallest, and the other two are widely distributed. We can also find that the coefficient of variation after simulating null models becomes larger, and null0 is particularly obvious. The correlation of the centrality series in all networks after simulating first order null models is low, and if we simulate higher order null model to the network, we can get more similar network as the original network, so the correlation of the closeness centrality series becomes large.

4. Conclusions and discussions

We analyzed the impact of variable network topology on the four centralities. The variable network topology was centralized on tunable clustering and assortative coefficient respectively. We first analyzed the gini index, coefficient of variation of the four centralities in the scale-free networks with adjustable cluster coefficients under different parameters. It was found that although the gini coefficient and coefficient of variation of closeness and betweenness increased with the increase of cluster coefficient, the gini coefficient and coefficient of variation of the degree and eigenvector centralities were less affected by the increase of the cluster coefficient. Then we further maintained the cluster and power law characteristics of the network, and then adjusted the assortative correlation of the network. The assortative correlation of the network was from the negative to the positive, the gini coefficient and the coefficient of variation of degree and betweenness increased. While the closeness and eigenvector were reversed.

Finally, we studied effects of null models of real networks. We used 10 real networks with different network topology and three null models to simulate these ten networks. After simulating three null models, the

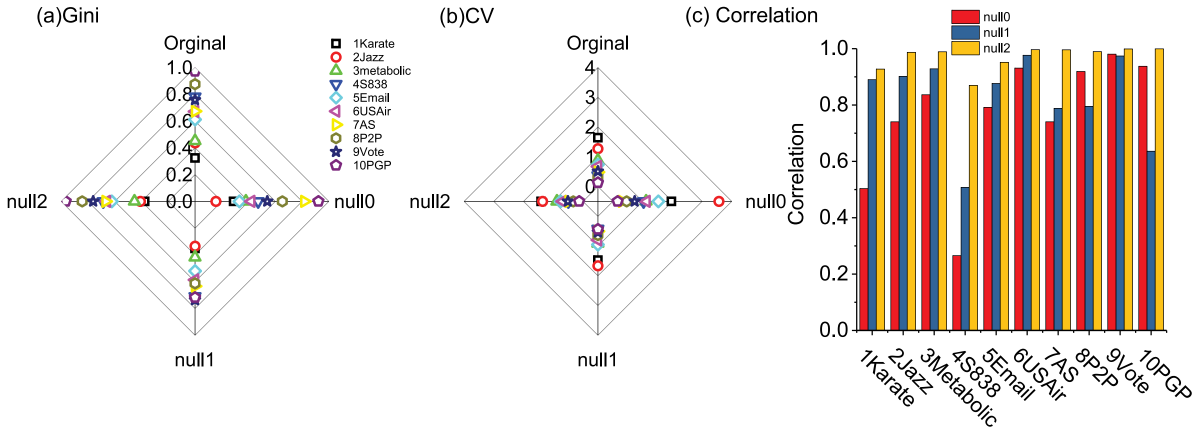

Fig. 8. Variation of eigenvector centrality in 10 networks after simulating three null models. We compare the eigenvector centrality between original networks and the randomization of networks by three orders of null models. (a) The gini value range of the eigenvector centrality in the original network and the three null models are relatively wide. (b) Compared with the coefficient of variation in the original networks, three null models in the networks seem unchanged much, except the Jazz network. (c) Correlation of eigenvector centrality between the original networks and three null models.

clustering and assortative coefficient of the network changed greatly. We also analyzed the gini index and coefficient of variation of the four centralities. We found that the gini coefficient of closeness was the smallest, betweenness was the largest, and the other two were widely distributed. After the null models, the gini index of the four centralities could became smaller, especially after null0, this trend was most obvious. The coefficient of variation was just the opposite. We could observe that the coefficient of variation of closeness was the largest, betweenness was the smallest, and the other two were widely distributed. The coefficient of variation after simulating null models become larger, and null0 was particularly obvious. In addition, we also gave that the correlation of the node centrality series before and after changing the clustering and assortativity. As the change of clustering and assortativity coefficient increased, the correlation of the centrality also became low.

Through these, the effect of varying topology on node centrality was great in complex networks. Thereupon we suggest that the traditional centrality methods are by no means easy to apply to identify significant or influential nodes, or rank the nodes in the networks without considering network topology. Our results might also find practical applications in optimized immunization strategies, which can be designed to monitor the actual spreaders. In a broader context, our work could be relevant to other fields of spreading processes, such as information, behavior, rumor spreading or other dynamical processes, which may provide insights into the analysis of the collective behavior, from social influence to biomedical responses.

CRediT authorship contribution statement

Xing-Zhang Wen: Conceptualization, Methodology, Validation, Formal analysis, Writing. **Yue Zheng:** Formal analysis, Writing. **Wen-Li Du:** Formal analysis, Writing. **Zhuo-Ming Ren:** Conceptualization, Methodology, Validation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

The work is partially supported by Natural Science Foundation of Zhejiang Province (Grant Nos. LY21F030019),National Natural Science Foundation of China (Grant Nos. 61803137). All authors have read and agreed to the published version of the manuscript.

References

[1] Ercseyravasz M, Lichtenwalter RN, Chawla NV, Toroczkai Z. Range-limited centrality measures in complex networks. Phys Rev E 2012;85(6):066103.
[2] Martin T, Zhang X, Newman MEJ. Localization and centrality in networks. Phys Rev E 2014;90(5):052808.
[3] Sabidussi G. The centrality index of a graph. Psychometrika 1966;31(4):581–603.
[4] Goh K-I, Oh E, Kahng B, Kim D. Betweenness centrality correlation in social networks. Phys Rev E 2003;67(1):017101.
[5] Borgatti SP. Centrality and network flow. Social Networks 2005;27(1):55–71.
[6] Kitsak M, Gallos LK, Havlin S, Liljeros F, Muchnik L, Stanley HE, et al. Identification of influential spreaders in complex networks. Nat Phys 2010;6(11):888–93.
[7] Ugander J, Backstrom L, Marlow C, Kleinberg J. Structural diversity in social contagion. Proc Natl Acad Sci 2012;109(16):5962–6.
[8] Zhong L-F, Liu Q-H, Wang W, Cai S-M. Comprehensive influence of local and global characteristics on identifying the influential nodes. Physica A 2018;511:78–84.
[9] Travencolo BAN, Costa LDF. Accessibility in complex networks. Phys Lett A 2008;373(1):89–95.
[10] Dangalchev C. Residual closeness in networks. Physica A 2006;365(2):556–64.
[11] Zhang J, Xu X-K, Li P, Zhang K, Small M. Node importance for dynamical process on networks: A multiscale characterization. Chaos 2011;21(1):016107.
[12] Poulin R, Boily M-C, Mâsse BR. Dynamical systems to define centrality in social networks. Social Networks 2000;22(3):187–220.
[13] Zhong L-F, Shang M-S, Chen X-L, Cai S-M. Identifying the influential nodes via eigen-centrality from the differences and similarities of structure. Physica A 2018;510:77–82.
[14] Lu L, Chen D, Ren X, Zhang Q, Zhang Y, Zhou T. Vital nodes identification in complex networks. Phys Rep 2016;650(650):1–63.
[15] Liao H, Mariani MS, Medo M, Zhang Y, Zhou M. Ranking in evolving complex networks. Phys Rep 2017;689:1–54.
[16] Ren Z-M. Node influence of the dynamic networks. Acta Phys Sin 2020;69(4):048901.
[17] Ren Z, Zeng A, Zhang Y. Structure-oriented prediction in complex networks. Phys Rep 2018;750:1–51.
[18] Aral S, Walker D. Identifying influential and susceptible members of social networks. Science 2012;337(6092):337–41.
[19] Klemm K, Serrano MA, Eguiluz VM, Miguel MS. A measure of individual role in collective dynamics. Sci Rep 2012;2(1):292.
[20] Watts DJ, Strogatz SH. Collective dynamics of 'small-world' networks. Nature 1998;393(6684):440–2.
[21] Centola D. The spread of behavior in an online social network experiment. Science 2010;329(5996):1194–7.
[22] Bond RM, Fariss CJ, Jones JJ, Kramer ADI, Marlow C, Settle JE, et al. A 61-million-person experiment in social influence and political mobilization. Nature 2012;489(7415):295–8.

[23] Newman MEJ. Assortative mixing in networks. Phys Rev Lett 2002;89(20):208701.
[24] Foster JG, Foster DV, Grassberger P, Paczuski M. Edge direction and the structure of networks. Proc Natl Acad Sci USA 2010;107(24):10815–20.
[25] Mahadevan P, Hubble C, Krioukov D, Huffaker B, Vahdat A. Orbis: Rescaling degree correlations to generate annotated internet topologies. SIGCOMM Comput Commun Rev 2007;37(4):325–36.
[26] Maslov S, Sneppen K. Specificity and stability in topology of protein networks. Science 2002;296(5569):910–3.
[27] Ren Z-M, Pan X, Zhang Y-C. Significance of the nested structure in multiplex world trade networks. Complexity 2020;2020:8827840.
[28] Ren Z, Mariani MS, Zhang Y, Medo M. Randomizing growing networks with a time-respecting zero model. Phys Rev E 2018;97(5):052311.
[29] Cui W, Shang K, Zhang Y, Xiao J, Xu X. Constructing zero networks for community detection in complex networks. Eur Phys J B 2018;91(7):1–9.
[30] Milo R, Shenorr SS, Itzkovitz S, Kashtan N, Chklovskii DB, Alon U. Network motifs: Simple building blocks of complex networks. Science 2002;298(5594):824–7.
[31] Milo R, Itzkovitz S, Kashtan N, Levitt R, Shenorr SS, Ayzenshtat I, et al. Superfamilies of evolved and designed networks. Science 2004;303(5663):1538–42.
[32] Maslov S, Sneppen K, Zaliznyak A. Detection of topological patterns in complex networks: correlation profile of the internet. Physica A 2004;333:529–40.
[33] Xu X, Zhang J, Sun J, Small M. Revising the simple measures of assortativity in complex networks. Phys Rev E 2009;80(5):056106.
[34] Holme P, Kim BJ. Growing scale-free networks with tunable clustering. Phys Rev E 2002;65(2):026107.
[35] Barabasi A, Albert R. Emergence of scaling in random networks. Science 1999;286(5439):509–12.
[36] Guo Q, Zhou T, Liu J-G, Bai W-J, Wang B-H, Zhao M. Growing scale-free small-world networks with tunable assortative coefficient. Physica A 2006;371(2):814–22.
[37] Zachary WW. An information flow model for conflict and fission in small groups. J Anthropol Res 1977;33(4):452–73.
[38] Leskovec J, Huttenlocher DP, Kleinberg J. Predicting positive and negative links in online social networks. Assoc Comput Mach 2010;10:641–50.
[39] Boguñá M, Pastor-Satorras R, Díaz-Guilera A, Arenas A. Models of social networks based on social distance attachment. Phys Rev E 2004;70:056122.
[40] Guimera R, Danon L, Diazguilera A, Giralt F, Arenas A. Self-similar community structure in a network of human interactions. Phys Rev E 2003;68(6):065103.
[41] Duch J, Arenas A. Community detection in complex networks using extremal optimization. Phys Rev E 2005;72(2):027104.
[42] Colizza V, Pastor-Satorras R, Vespignani A. Reaction–diffusion processes and metapopulation models in heterogeneous networks. Nat Phys 2007;3(4):276–82.
[43] Leskovec J, Kleinberg J, Faloutsos C. Graphs over time: densification laws, shrinking diameters and possible explanations. Assoc Comput Mach 2005;KDD '05(11):177–87.
[44] Gleiser PM, Danon L. Community structure in Jazz. Adv Complex Syst 2003;06(04):565–73.